

# Effects of spin-orbit coupling on the BKT transition and the vortex- antivortex structure in 2D Fermi Gases

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**QMath13**

**Mathematical Results in Quantum Physics**

**Atlanta: October 10<sup>th</sup>, 2016**

# Main References for Talk

Ultra-cold fermions in the flatland: evolution from BCS to Bose superfluidity in two-dimensions with spin-orbit and Zeeman fields

Li Han and C. A. R. Sá de Melo

*School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA*

(Dated: June 22, 2012)

arXiv:1206.4984v1 (UNPUBLISHED)

PRL **113**, 165304 (2014)

PHYSICAL REVIEW LETTERS

week ending  
17 OCTOBER 2014

**Effects of Spin-Orbit Coupling on the Berezinskii-Kosterlitz-Thouless Transition and the Vortex-Antivortex Structure in Two-Dimensional Fermi Gases**

Jeroen P. A. Devreese,<sup>1,2</sup> Jacques Tempere,<sup>2,3</sup> and Carlos A. R. Sá de Melo<sup>1</sup>

<sup>1</sup>*School of Physics, Georgia Institute of Technology, Atlanta 30332, USA*

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Li Han



Ian Spielman



Jeroen Devreese



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# Outline

- 1) Introduction to 2D Fermi gases.
- 2) Creation of artificial spin-orbit coupling (SOC).
- 3) Quantum phases and topological quantum phase transitions of 2D Fermi gases with SOC.
- 4) The BKT transition and the vortex-antivortex structure.
- 5) Conclusions

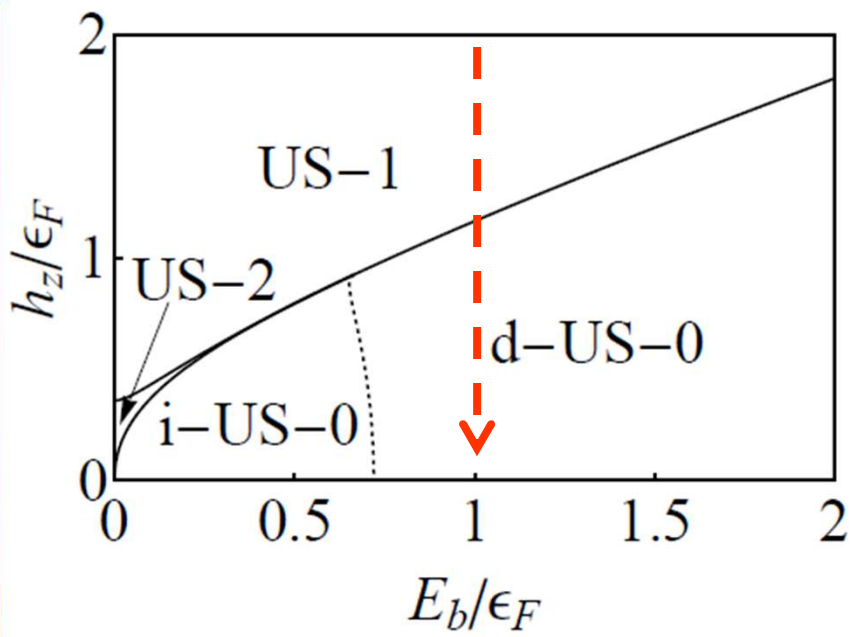
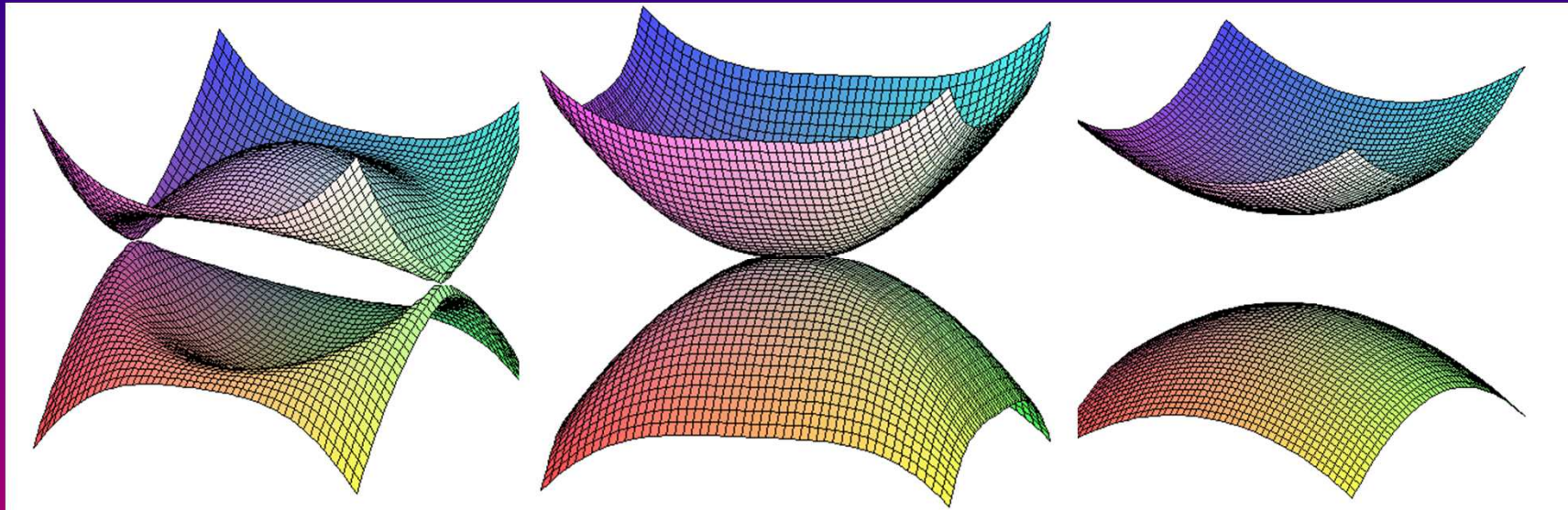
# Conclusions in words

- Ultra-cold fermions in the presence of spin-orbit and Zeeman fields are special systems that allow for the study of exciting new phases of matter, such as topological superfluids, with a high degree of accuracy.
- Topological quantum phase transitions emerge as function of Zeeman fields and binding energy for fixed spin-orbit coupling.

# Conclusions in words

- The critical temperature of the BKT transition as a function of pair binding energy is affected by the presence of spin-orbit effects and Zeeman fields. While the Zeeman field tends to reduce the critical temperature, SOC tends to stabilize it by introducing a triplet component in the superfluid order parameter.
- In the presence of a generic SOC the sound velocity in the superfluid state is anisotropic and becomes a sensitive probe of the proximity to topological quantum phase transitions. The vortex and antivortex shapes are also affected by the SOC and acquire a corresponding anisotropy.

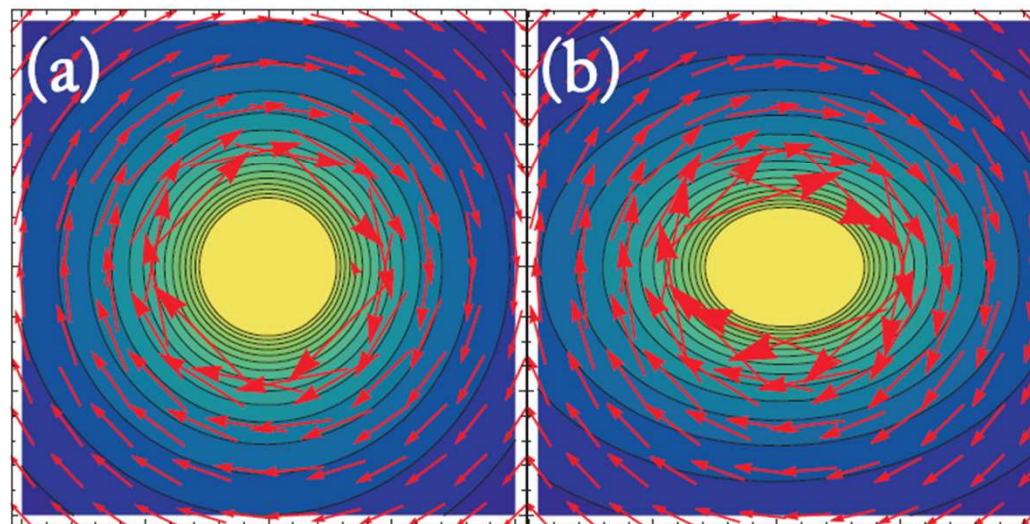
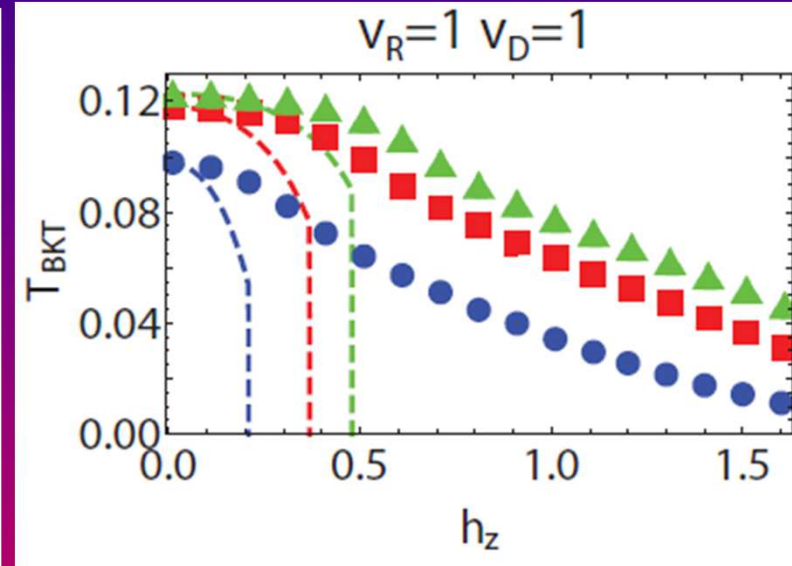
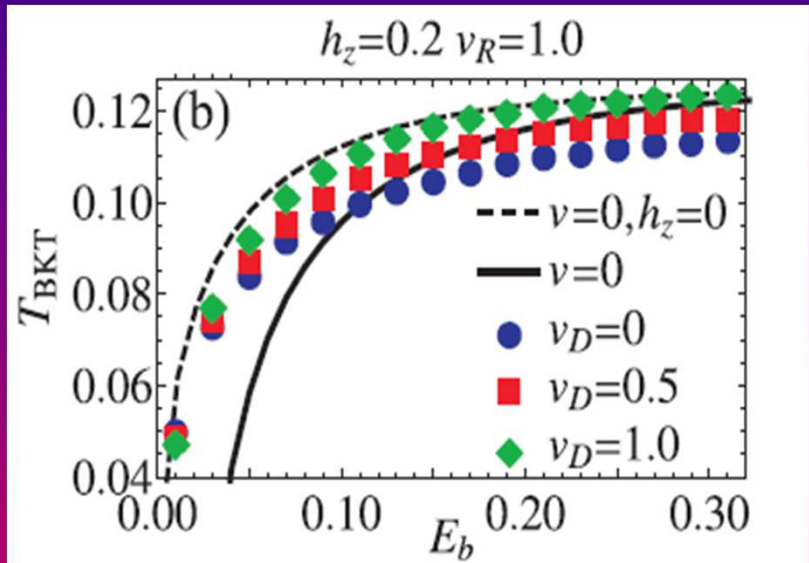
# Conclusions in Pictures



**Change in topology**

**TRANSITION FROM  
GAPLESS TO GAPPED SUPERFLUID**

# BKT transition and vortex-antivortex structure



Rashba

ERD



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1) Introduction to 2D Fermi gases.

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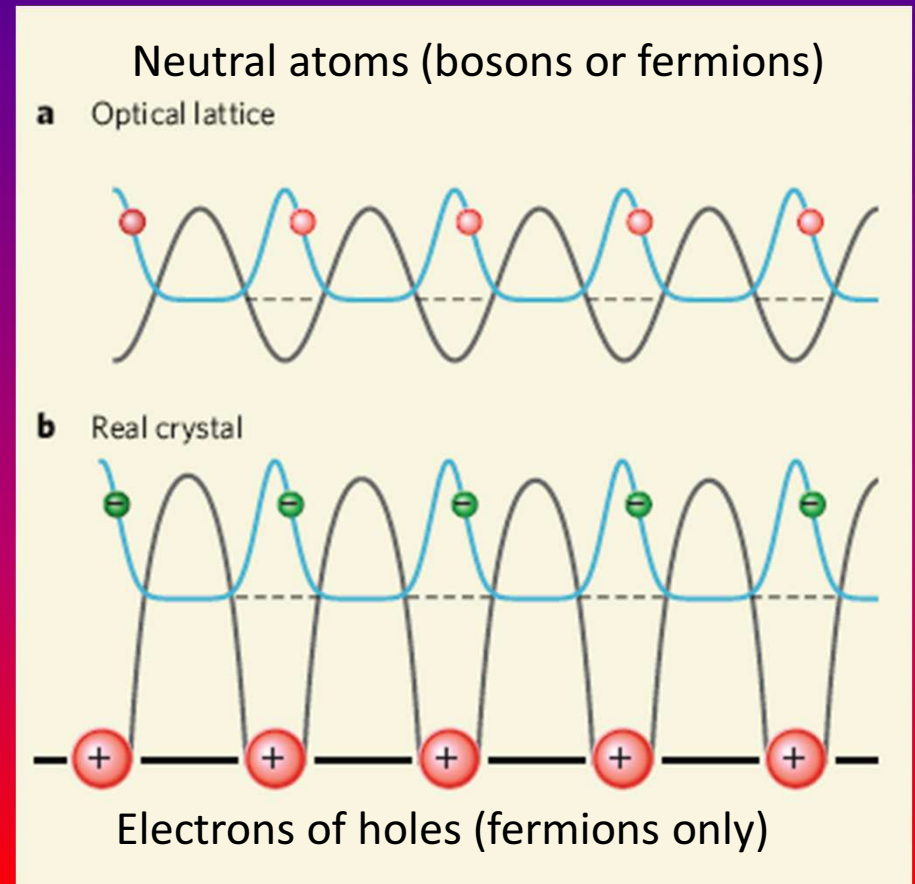
4) The BKT transition and the vortex-antivortex structure.

5) Conclusions

# Condensed Matter meets Atomic Physics

In optical lattices many types of atoms can be loaded like bosonic, Sodium-23, Potassium-39, Rubidium-87, or Cesium-133; and fermionic Lithium-6, Potassium-40, Strontium-87, etc...

In real crystals electrons or holes (absence of electrons) may be responsible for many “electronic” phases of condensed matter physics, such as metallic, insulating, superconducting, ferromagnetic, anti-ferromagnetic, etc...



# How atoms are trapped?

- Atom-laser interaction
- Induced dipole moment.
- Trapping potential

$$V(\mathbf{r}, t) = -\mathbf{d} \cdot \mathbf{E}(\mathbf{r}, t)$$

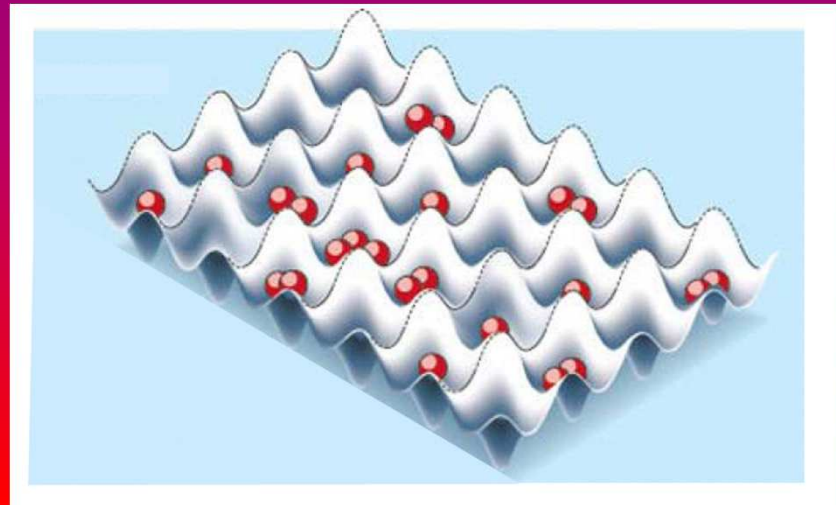
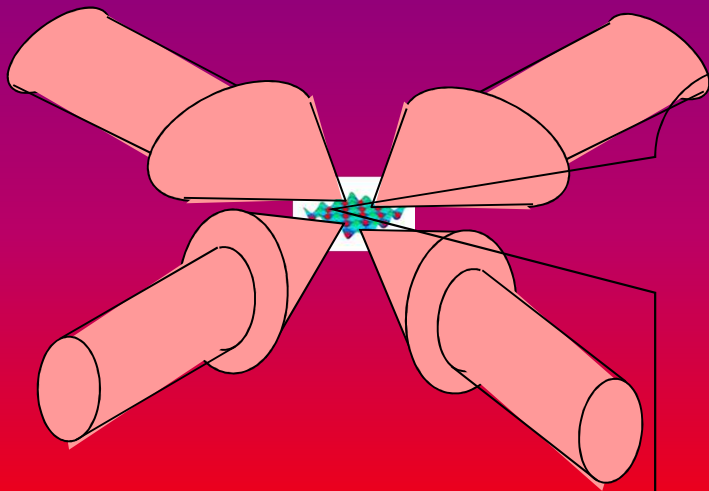
$$\mathbf{d} = -\alpha(\omega) \mathbf{E}(\mathbf{r}, t)$$

$$V(\mathbf{r}, t) = -\alpha(\omega) [\mathbf{E}(\mathbf{r}, t)]^2$$

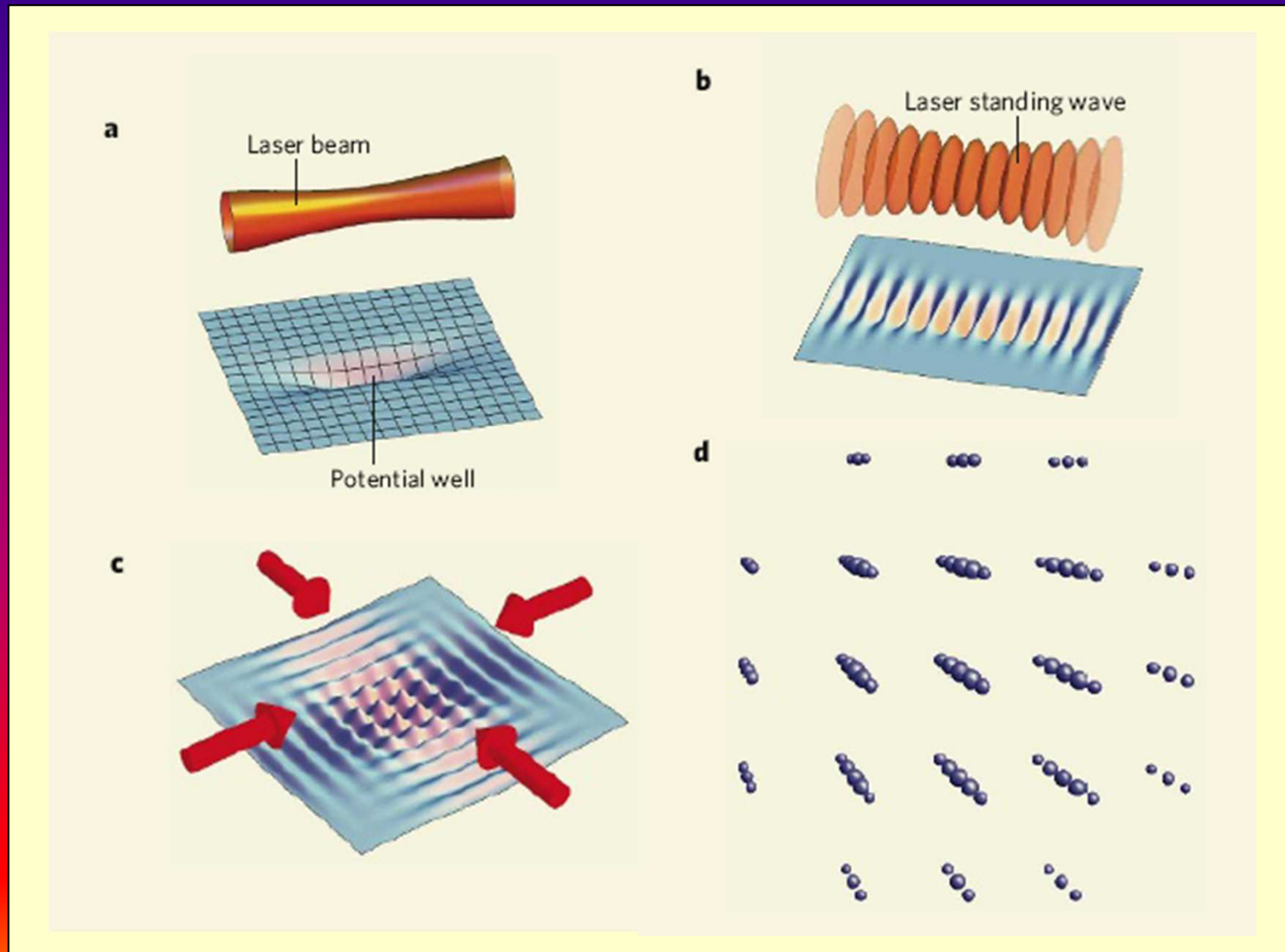
# Atoms in optical lattices

$$V(\mathbf{r}) = -\alpha(\omega) \langle [\mathbf{E}(\mathbf{r}, t)]^2 \rangle$$

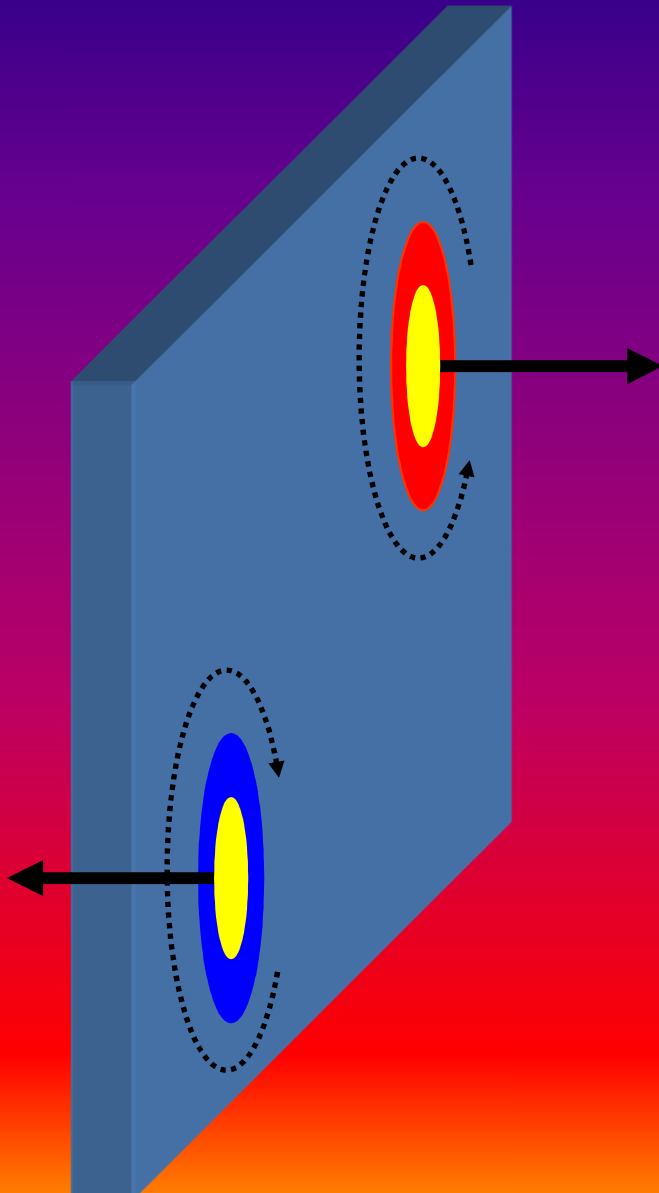
$$V(\mathbf{r}) = -\frac{1}{2} \alpha(\omega) [\mathbf{E}(\mathbf{r})]^2$$



# How optical lattices are created?



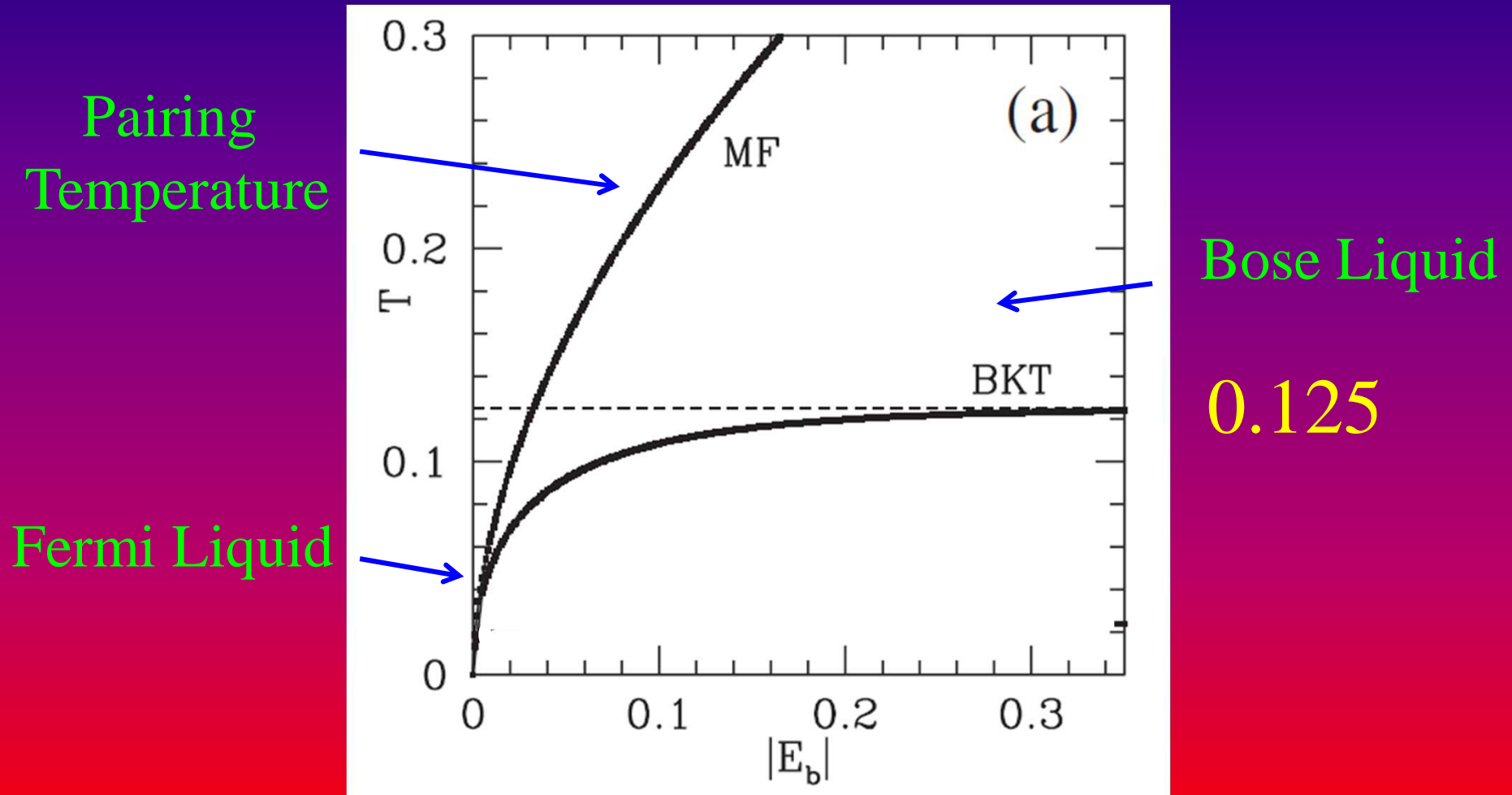
# Single plane excitations



Vortex-antivortex pairs

BKT transition:  
Physics of 2D XY model

# Critical Temperature



BCS-Bose Superfluidity in 2D

# 2D Fermi gases with increasing attractive interactions, but no SOC.

## Vortex-Antivortex Lattice in Ultracold Fermionic Gases

S. S. Botelho and C. A. R. Sá de Melo

*School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA*

(Received 14 September 2005; published 3 February 2006)

PRL **96**, 040404 (2006)

PRL **105**, 030404 (2010)

PHYSICAL REVIEW LETTERS

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16 JULY 2010



## Observation of a Two-Dimensional Fermi Gas of Atoms

Kirill Martiyanov, Vasiliy Makhalov, and Andrey Turlapov\*

*Institute of Applied Physics, Russian Academy of Sciences, ul. Ulyanova 46, Nizhniy Novgorod, 603000, Russia*

(Received 20 May 2010; published 15 July 2010)



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1) Introduction to 2D Fermi gases.

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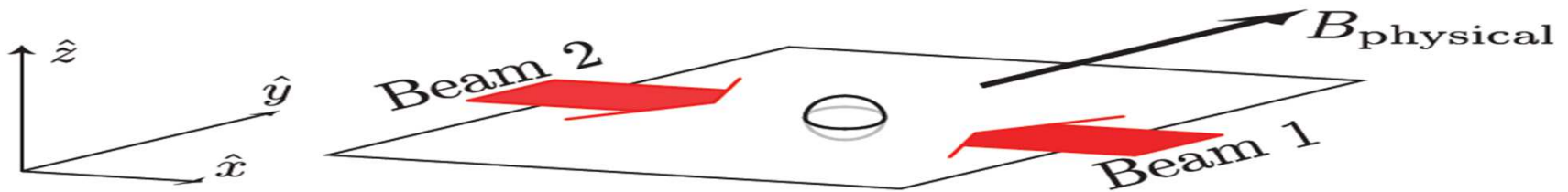
3) Quantum phases and topological quantum phase transitions of 2D Fermi gases with SOC.

4) The BKT Transition and the vortex-antivortex structure.

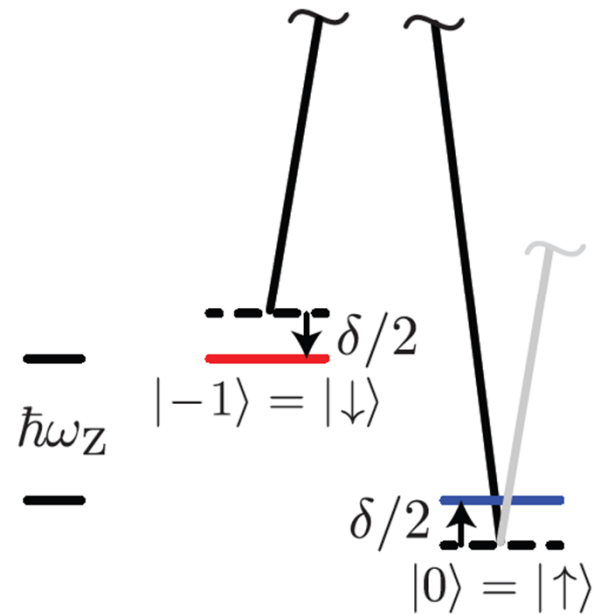
5) Conclusions

# Raman process and spin-orbit coupling

## Geometry



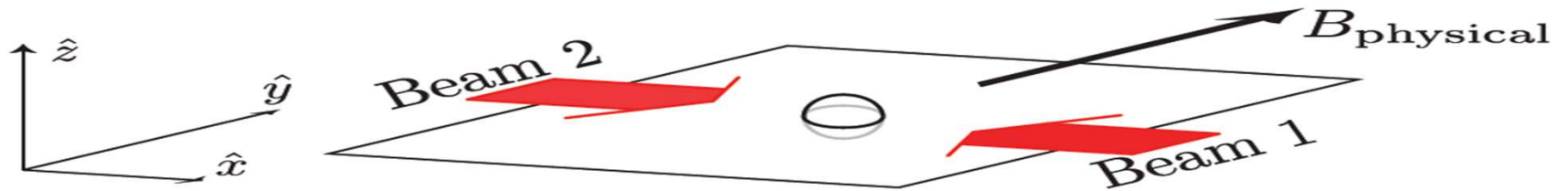
$$\begin{pmatrix} \frac{(\mathbf{k} - \mathbf{k}_R)^2}{2m} + \frac{\delta}{2} & \frac{\Omega}{2} \\ \frac{\Omega}{2} & \frac{(\mathbf{k} + \mathbf{k}_R)^2}{2m} - \frac{\delta}{2} \end{pmatrix}$$



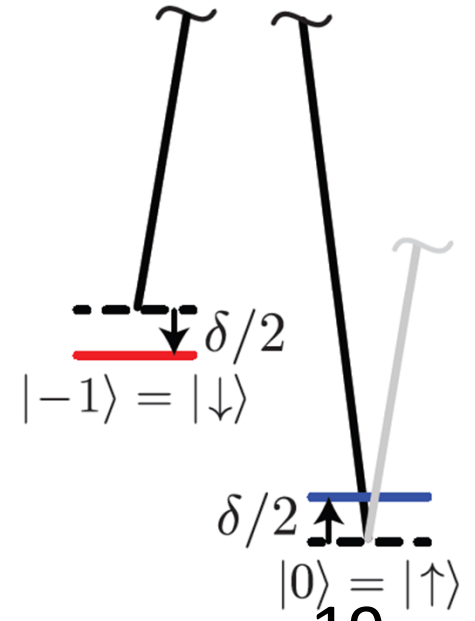
# SU(2) rotation to new spin basis:

$$\sigma_x \rightarrow \sigma_z; \sigma_z \rightarrow \sigma_y; \sigma_y \rightarrow \sigma_x$$

## Geometry



$$\begin{pmatrix} \frac{\mathbf{k}^2 + k_R^2}{2m} + \frac{\Omega}{2} & -i \left( \frac{\delta}{2} - \frac{k_R}{m} k_x \right) \\ i \left( \frac{\delta}{2} - \frac{k_R}{m} k_x \right) & \frac{\mathbf{k}^2 + k_R^2}{2m} - \frac{\Omega}{2} \end{pmatrix} \begin{matrix} - \\ \hbar\omega_Z \\ - \end{matrix}$$



## LETTER

doi:10.1038/nature09887

## Spin-orbit-coupled Bose-Einstein condensates

Y.-J. Lin<sup>1</sup>, K. Jiménez-García<sup>1,2</sup> & I. B. Spielman<sup>1</sup>

$$\begin{pmatrix} \frac{\mathbf{k}^2 + k_R^2}{2m} + \frac{\Omega}{2} & -i \left( \frac{\delta}{2} - \frac{k_R}{m} k_x \right) \\ i \left( \frac{\delta}{2} - \frac{k_R}{m} k_x \right) & \frac{\mathbf{k}^2 + k_R^2}{2m} - \frac{\Omega}{2} \end{pmatrix}$$

spin-orbit

detuning

Raman  
coupling

# Hamiltonian with spin-orbit

## Hamiltonian with spin - orbit

$$H = \sum_{k,s} \varepsilon(\mathbf{k}) c_{ks}^{\dagger} c_{ks} - \sum_{k,s} h_{s's}(\mathbf{k}) c_{ks'}^{\dagger} c_{ks}$$

# Parallel and perpendicular fields

$$h_{\parallel}(\mathbf{k}) = h_z(\mathbf{k})$$

$$h_{\perp}(\mathbf{k}) = h_x(\mathbf{k}) - ih_y(\mathbf{k})$$

$$\mathbf{H}_0(\mathbf{k}) = \begin{pmatrix} \varepsilon(\mathbf{k}) - h_{\parallel}(\mathbf{k}) & -h_{\perp}(\mathbf{k}) \\ -h_{\perp}^*(\mathbf{k}) & \varepsilon(\mathbf{k}) + h_{\parallel}(\mathbf{k}) \end{pmatrix}$$

# Hamiltonian in terms of $\mathbf{k}$ -dependent magnetic fields

## Hamiltonian Matrix

$$\mathbf{H}_0(\mathbf{k}) = \varepsilon(\mathbf{k})\mathbf{1} - h_x(\mathbf{k})\boldsymbol{\sigma}_x - h_y(\mathbf{k})\boldsymbol{\sigma}_y - h_z(\mathbf{k})\boldsymbol{\sigma}_z$$

Momentum Space Two - Level System  
in a momentum dependent magnetic field

$$\mathbf{h}(\mathbf{k}) = [h_x(\mathbf{k}), h_y(\mathbf{k}), h_z(\mathbf{k})]$$

# Eigenvalues

$$\varepsilon_{\uparrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) - |h_{\text{eff}}(\mathbf{k})|$$

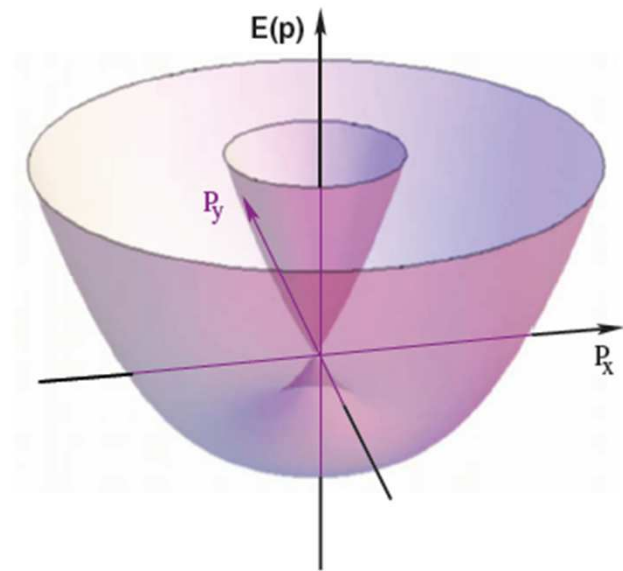
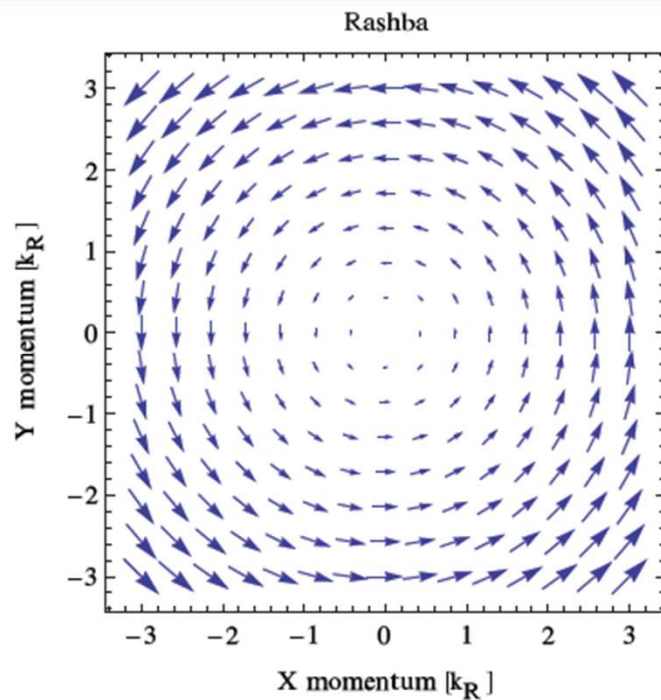
$$\varepsilon_{\downarrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) + |h_{\text{eff}}(\mathbf{k})|$$

$$|h_{\text{eff}}(\mathbf{k})| = \sqrt{|h_x(\mathbf{k})|^2 + |h_y(\mathbf{k})|^2 + |h_z(\mathbf{k})|^2}$$



# Rashba Spin-Orbit Coupling

$$\mathbf{H}_R(\mathbf{k}) = v_R \begin{pmatrix} 0 & k_y + ik_x \\ k_y - ik_x & 0 \end{pmatrix}$$

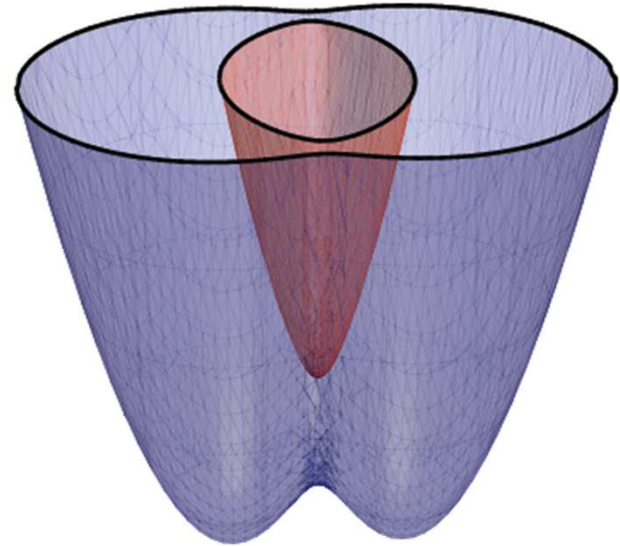
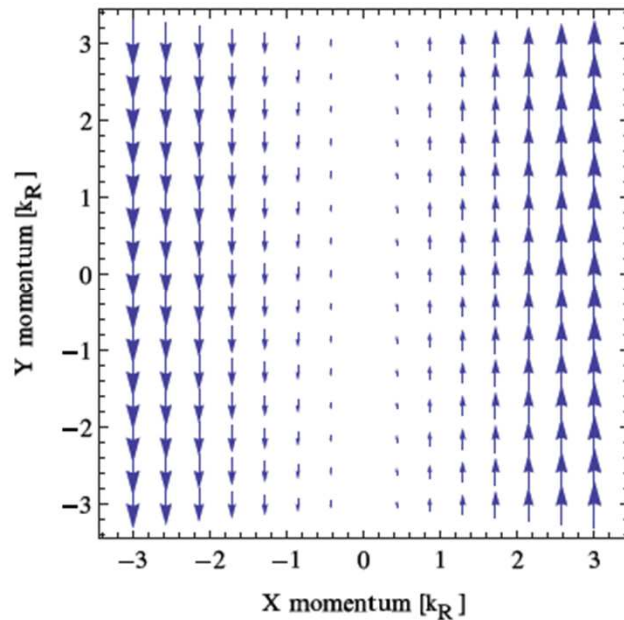


# Equal-Rashba-Dresselhaus (ERD) Spin-Orbit Coupling

$$\mathbf{H}_R(\mathbf{k}) = v_R \begin{pmatrix} 0 & k_y + ik_x \\ k_y - ik_x & 0 \end{pmatrix}$$

$$\mathbf{H}_D(\mathbf{k}) = -v_D \begin{pmatrix} 0 & k_y - ik_x \\ k_y + ik_x & 0 \end{pmatrix}$$

$$\mathbf{H}_{ERD}(\mathbf{k}) = v \begin{pmatrix} 0 & ik_x \\ -ik_x & 0 \end{pmatrix}$$



# Energy Dispersions in the ERD case

Simpler case :

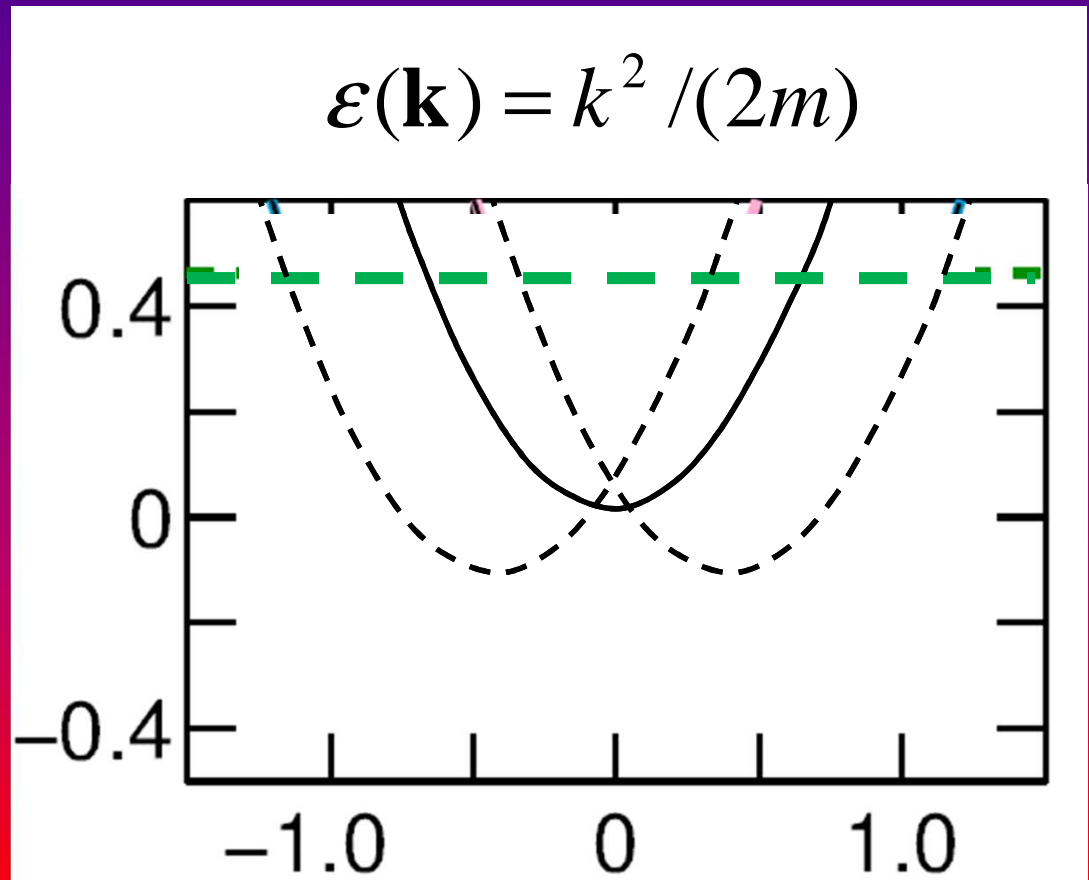
$$h_x(\mathbf{k}) = 0$$

$$h_y(\mathbf{k}) = vk_x$$

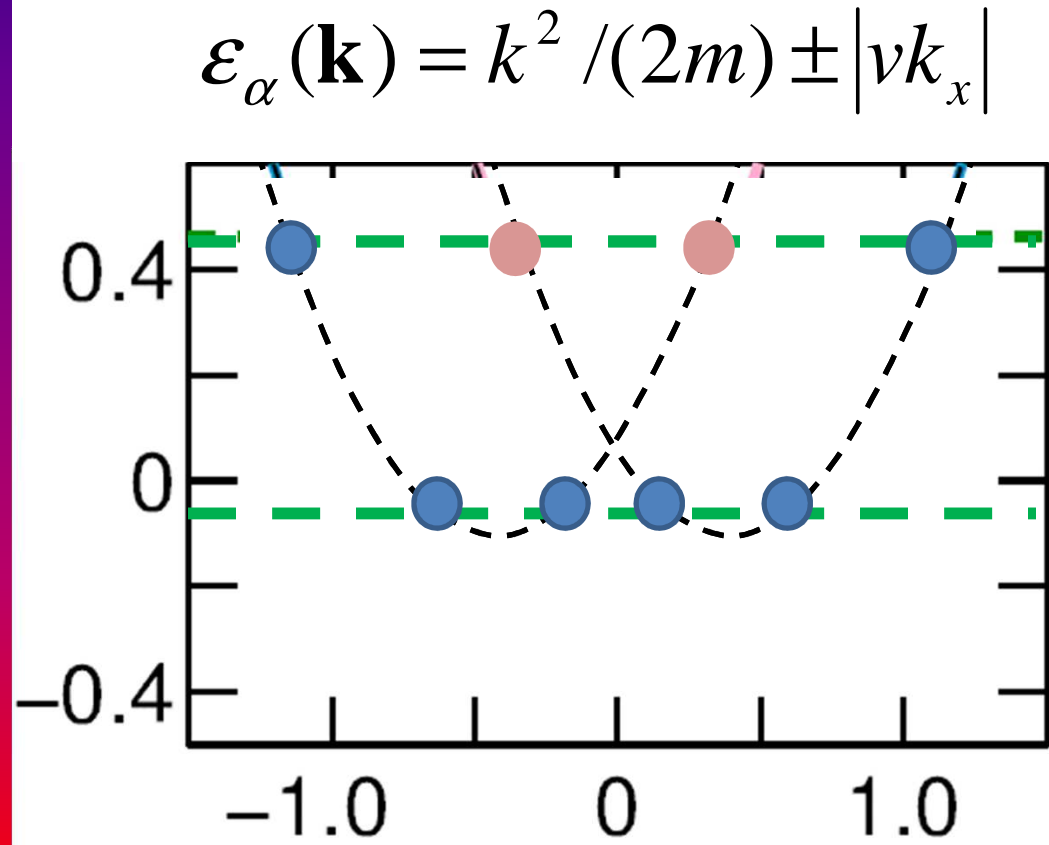
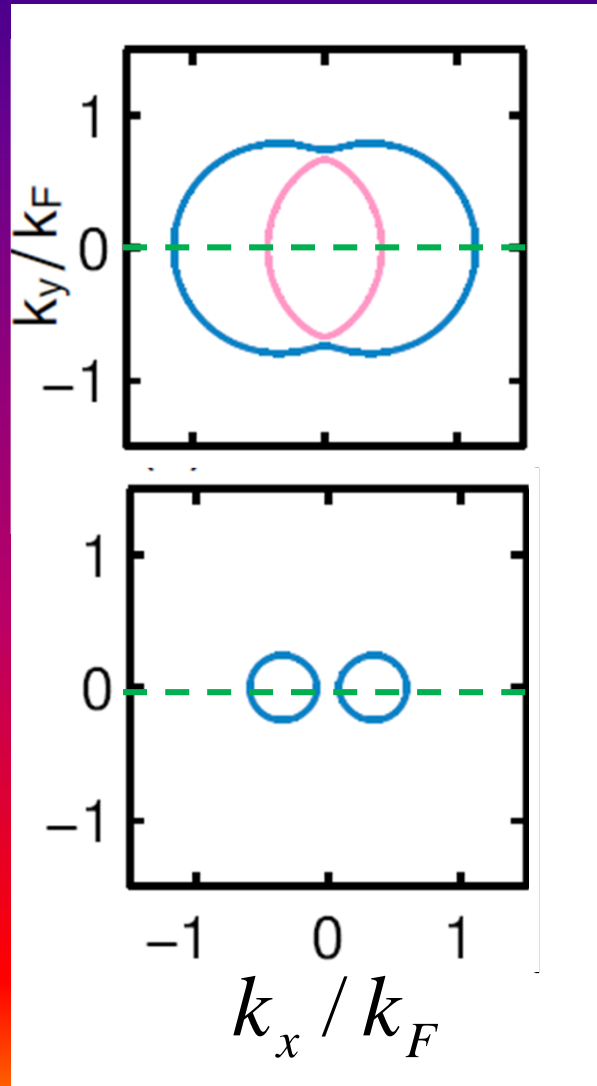
$$h_z(\mathbf{k}) = 0$$

$$\varepsilon_{\uparrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) - |vk_x|$$

$$\varepsilon_{\downarrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) + |vk_x|$$



# Energy Dispersions and Fermi Surfaces



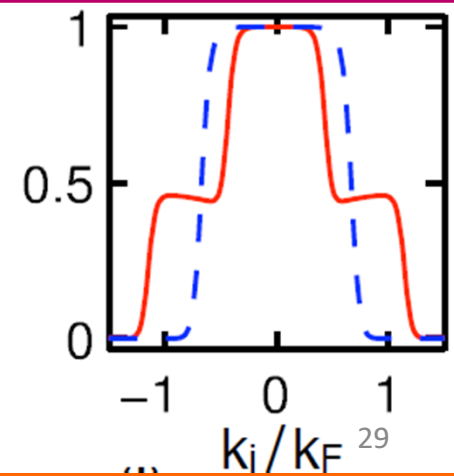
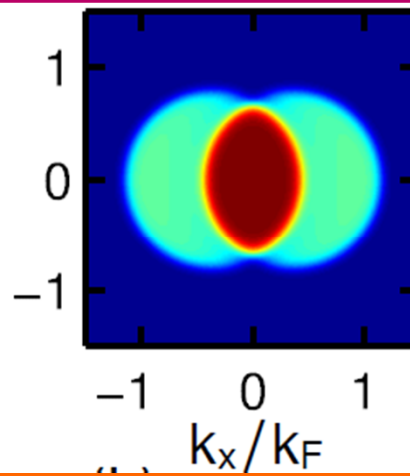
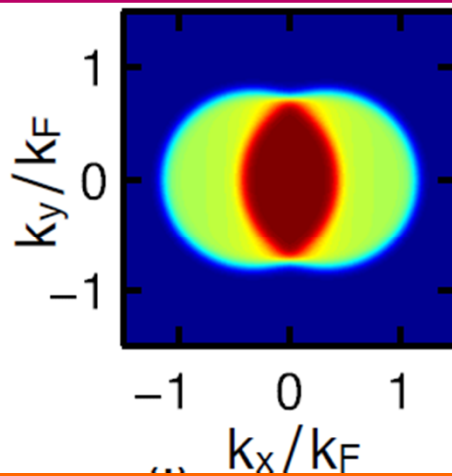
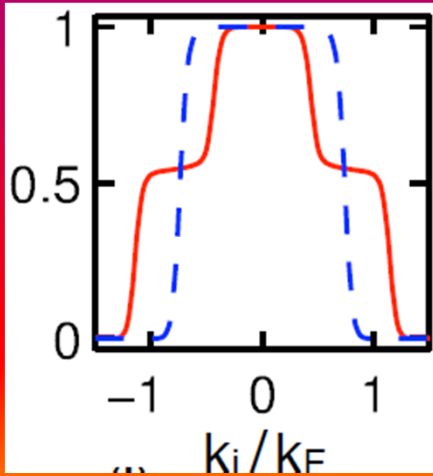
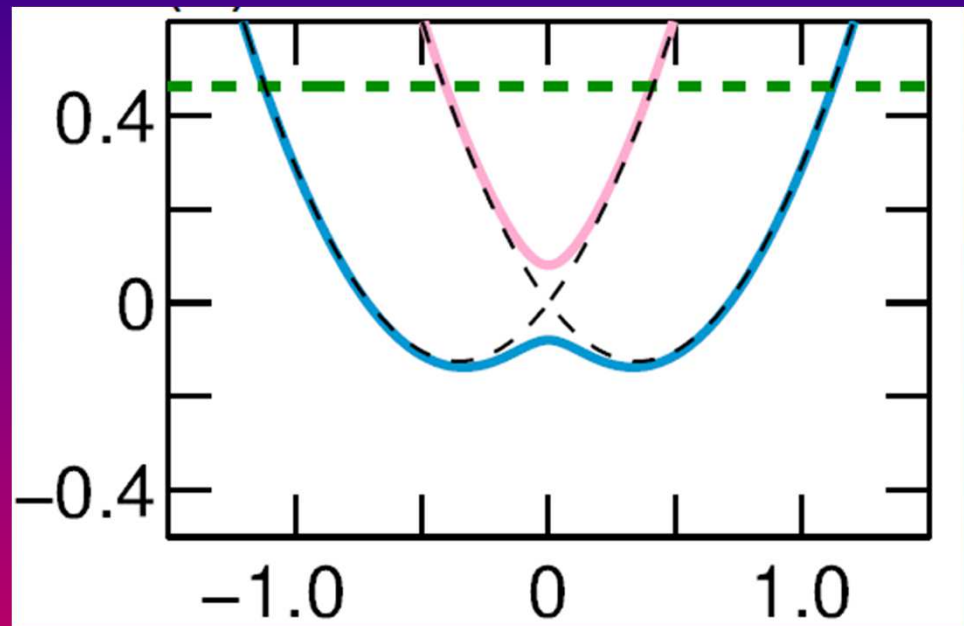
$$k_x / k_F$$

# Momentum Distribution (Parity)

$$h_x(\mathbf{k}) = 0$$

$$\frac{h_y(\mathbf{k})}{\mathcal{E}_F} = 0.71 \frac{k_x}{k_F}$$

$$\frac{h_z(\mathbf{k})}{\mathcal{E}_F} = 0.05$$



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# Bring Interactions Back (real space)

$$\mathcal{H}(\mathbf{r}) = \mathcal{H}_0(\mathbf{r}) + \mathcal{H}_I(\mathbf{r})$$

$$\mathcal{H}_0(\mathbf{r}) = \sum_{\alpha\beta} \psi_{\alpha}^{\dagger}(\mathbf{r}) \left[ \hat{K}_{\alpha} \delta_{\alpha\beta} - h_i(\mathbf{r}) \sigma_{i,\alpha\beta} \right] \psi_{\beta}(\mathbf{r})$$

Kinetic Energy

Spin-orbit and Zeeman

$$\mathcal{H}_I(\mathbf{r}) = -g \psi_{\uparrow}^{\dagger}(\mathbf{r}) \psi_{\downarrow}^{\dagger}(\mathbf{r}) \psi_{\downarrow}(\mathbf{r}) \psi_{\uparrow}(\mathbf{r})$$

Contact Interaction

# Bring Interactions Back (momentum space)

$$\mathcal{H}_I = -g \sum_{\mathbf{q}} b^\dagger(\mathbf{q}) b(\mathbf{q})$$

$$b^\dagger(\mathbf{q}) = \sum_{\mathbf{k}} \psi_{\uparrow}^\dagger(\mathbf{k} + \mathbf{q}/2) \psi_{\downarrow}^\dagger(-\mathbf{k} + \mathbf{q}/2)$$

$$\Delta_0 = -g \langle b(\mathbf{q} = 0) \rangle \quad \text{and} \quad \Delta_0^* = -g \langle b^\dagger(\mathbf{q} = 0) \rangle$$



# Bring interactions back: Hamiltonian in initial spin basis

$\psi_{k\uparrow}$	$\psi_{k\downarrow}$	$\psi_{-k\uparrow}^+$	$\psi_{-k\downarrow}^+$
--------------------	----------------------	-----------------------	-------------------------

$\psi_{k\uparrow}^+$	$\psi_{k\downarrow}^+$	$\psi_{-k\uparrow}$	$\psi_{-k\downarrow}$
----------------------	------------------------	---------------------	-----------------------

$$\mathbf{H}_0 = \begin{pmatrix} \tilde{K}_{\uparrow}(\mathbf{k}) & -h_{\perp}(\mathbf{k}) & 0 & -\Delta_0 \\ -h_{\perp}^*(\mathbf{k}) & \tilde{K}_{\downarrow}(\mathbf{k}) & \Delta_0 & 0 \\ 0 & \Delta_0^{\dagger} & -\tilde{K}_{\uparrow}(-\mathbf{k}) & h_{\perp}^*(-\mathbf{k}) \\ -\Delta_0^{\dagger} & 0 & h_{\perp}(-\mathbf{k}) & -\tilde{K}_{\downarrow}(-\mathbf{k}) \end{pmatrix}$$

$$\tilde{K}_s(\mathbf{k}) = \varepsilon(\mathbf{k}) - \mu - sh_z(\mathbf{k})$$

# Bring interactions back: Hamiltonian in the generalized helicity basis

$$\Phi_{k\uparrow}$$

$$\Phi_{k\downarrow}$$

$$\Phi_{-k\uparrow}^+$$

$$\Phi_{-k\downarrow}^+$$

$\Phi_{k\uparrow}^+$
$\Phi_{k\downarrow}^+$
$\Phi_{-k\uparrow}$
$\Phi_{-k\downarrow}$

$$\tilde{\mathbf{H}}_0 = \begin{pmatrix} \xi_{\uparrow}(\mathbf{k}) & 0 & \Delta_T(\mathbf{k})e^{-i\varphi_{\mathbf{k}}} & -\Delta_S(\mathbf{k}) \\ 0 & \xi_{\downarrow}(\mathbf{k}) & \Delta_S(\mathbf{k}) & -\Delta_T e^{i\varphi_{\mathbf{k}}} \\ \Delta_T^*(\mathbf{k})e^{i\varphi_{\mathbf{k}}} & -\Delta_S^*(\mathbf{k}) & -\xi_{\uparrow}(\mathbf{k}) & 0 \\ \Delta_S^*(\mathbf{k}) & -\Delta_T^*(\mathbf{k})e^{-i\varphi_{\mathbf{k}}} & 0 & -\xi_{\downarrow}(\mathbf{k}) \end{pmatrix}$$

$$\varphi_{\mathbf{k}} = \text{Arg} [h_{\perp}(\mathbf{k})]$$

# Order Parameter: Singlet & Triplet

$$\Delta_S(\mathbf{k}) = \Delta_0 h_{\parallel}(\mathbf{k}) / |\mathbf{h}_{\text{eff}}(\mathbf{k})|$$

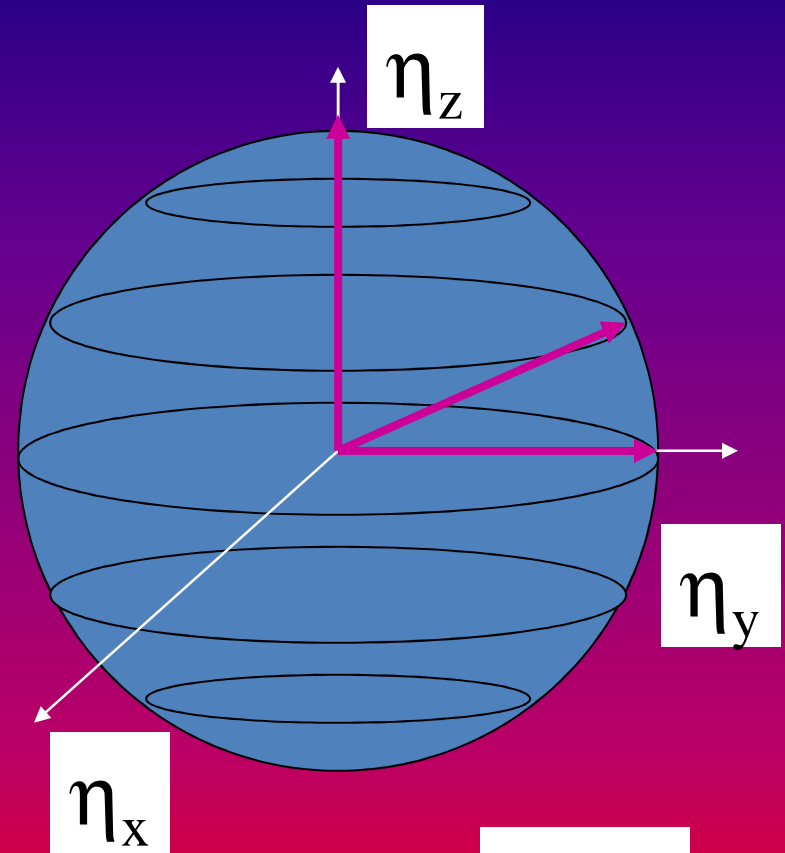
$$\Delta_T(\mathbf{k}) = \Delta_0 |h_{\perp}(\mathbf{k})| / |\mathbf{h}_{\text{eff}}(\mathbf{k})|$$

$$|\Delta_T(\mathbf{k})|^2 + |\Delta_S(\mathbf{k})|^2 = |\Delta_0|^2$$

$$h_{\perp}(\mathbf{k}) = vk_x \quad h_z(\mathbf{k}) = h_z$$

$$\mathbf{h}_{\text{eff}}(\mathbf{k}) = (0, vk_x, h_z)$$

$$h_{\text{eff}}(\mathbf{k}) = \sqrt{|vk_x|^2 + h_z^2}$$



ERD

# Excitation Spectrum

$$E_1(\mathbf{k}) = \sqrt{\left[ \left( \frac{\xi_{\uparrow} - \xi_{\downarrow}}{2} \right) - \sqrt{\left( \frac{\xi_{\uparrow} + \xi_{\downarrow}}{2} \right)^2 + |\Delta_S(\mathbf{k})|^2} \right]^2 + |\Delta_T(\mathbf{k})|^2},$$

$$E_2(\mathbf{k}) = \sqrt{\left[ \left( \frac{\xi_{\uparrow} - \xi_{\downarrow}}{2} \right) + \sqrt{\left( \frac{\xi_{\uparrow} + \xi_{\downarrow}}{2} \right)^2 + |\Delta_S(\mathbf{k})|^2} \right]^2 + |\Delta_T(\mathbf{k})|^2},$$

Can be  
zero

$$E_3(\mathbf{k}) = -E_2(\mathbf{k})$$

$$\xi_{\uparrow}(\mathbf{k}) = K_+(\mathbf{k}) - |\mathbf{h}_{\text{eff}}(\mathbf{k})|$$

$$E_4(\mathbf{k}) = -E_1(\mathbf{k})$$

$$\xi_{\downarrow}(\mathbf{k}) = K_+(\mathbf{k}) + |\mathbf{h}_{\text{eff}}(\mathbf{k})|$$

# Excitation Spectrum

Making singlet and triplet sectors explicit

$$E_2(\mathbf{k}) \leftrightarrow E_-(\mathbf{k})$$

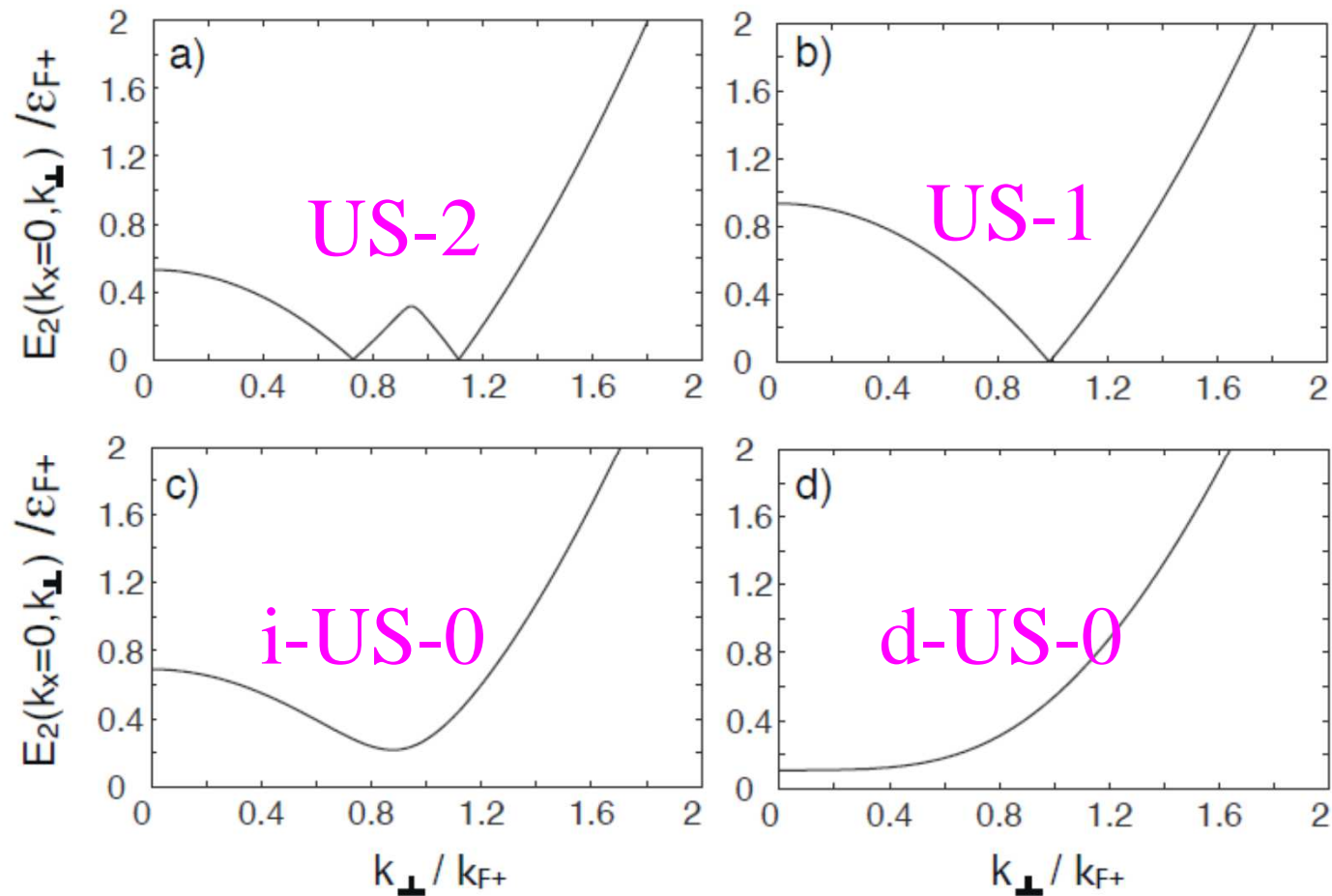
$$E_1(\mathbf{k}) \leftrightarrow E_+(\mathbf{k})$$

$$E_{p\pm}(\mathbf{k}) = \sqrt{(E_S(\mathbf{k}) \pm |\mathbf{h}_{\text{eff}}(\mathbf{k})|)^2 + |\Delta_T(\mathbf{k})|^2},$$

$$E_S(\mathbf{k}) = \sqrt{|K(\mathbf{k})|^2 + |\tilde{\Delta}_S(\mathbf{k})|^2}$$

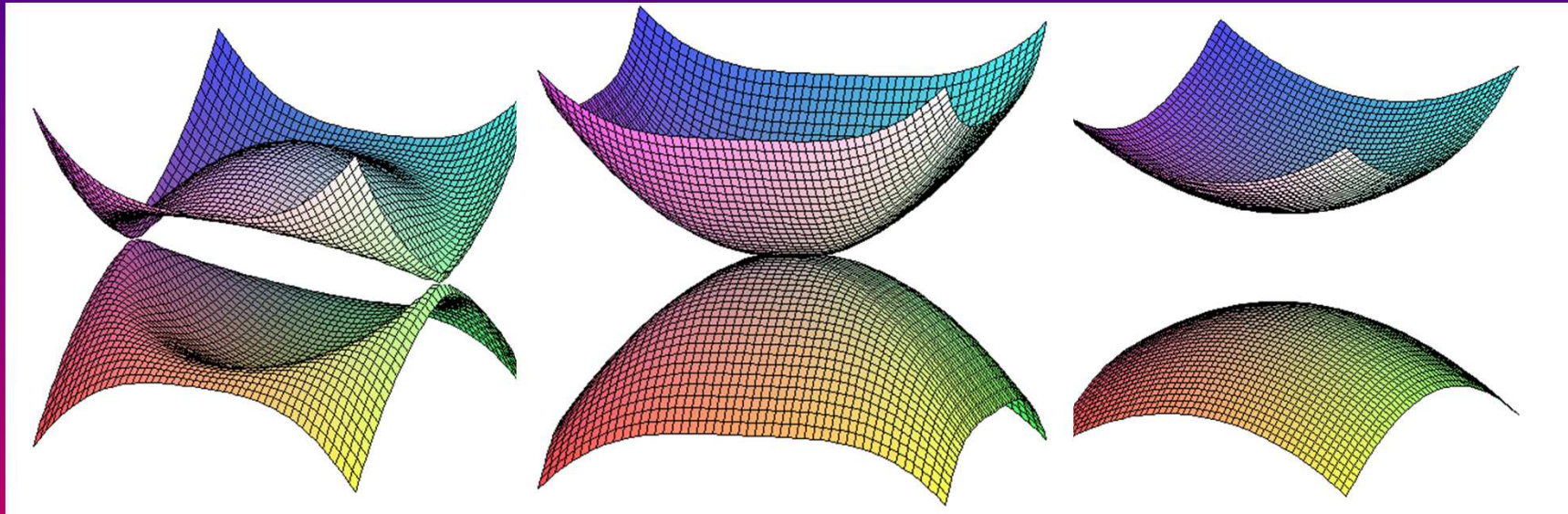
singlet sector

# Excitation Spectrum (ERD)



$$\Delta_T(\mathbf{k}) = \Delta_0 |h_{\perp}(\mathbf{k})| / |\mathbf{h}_{\text{eff}}(\mathbf{k})| = 0$$

# Lifshitz transition



**Change in topology**

# Topological invariant (charge) in 2D

$$\hat{\mathbf{m}}(\mathbf{k}) = (m_x, m_y)$$

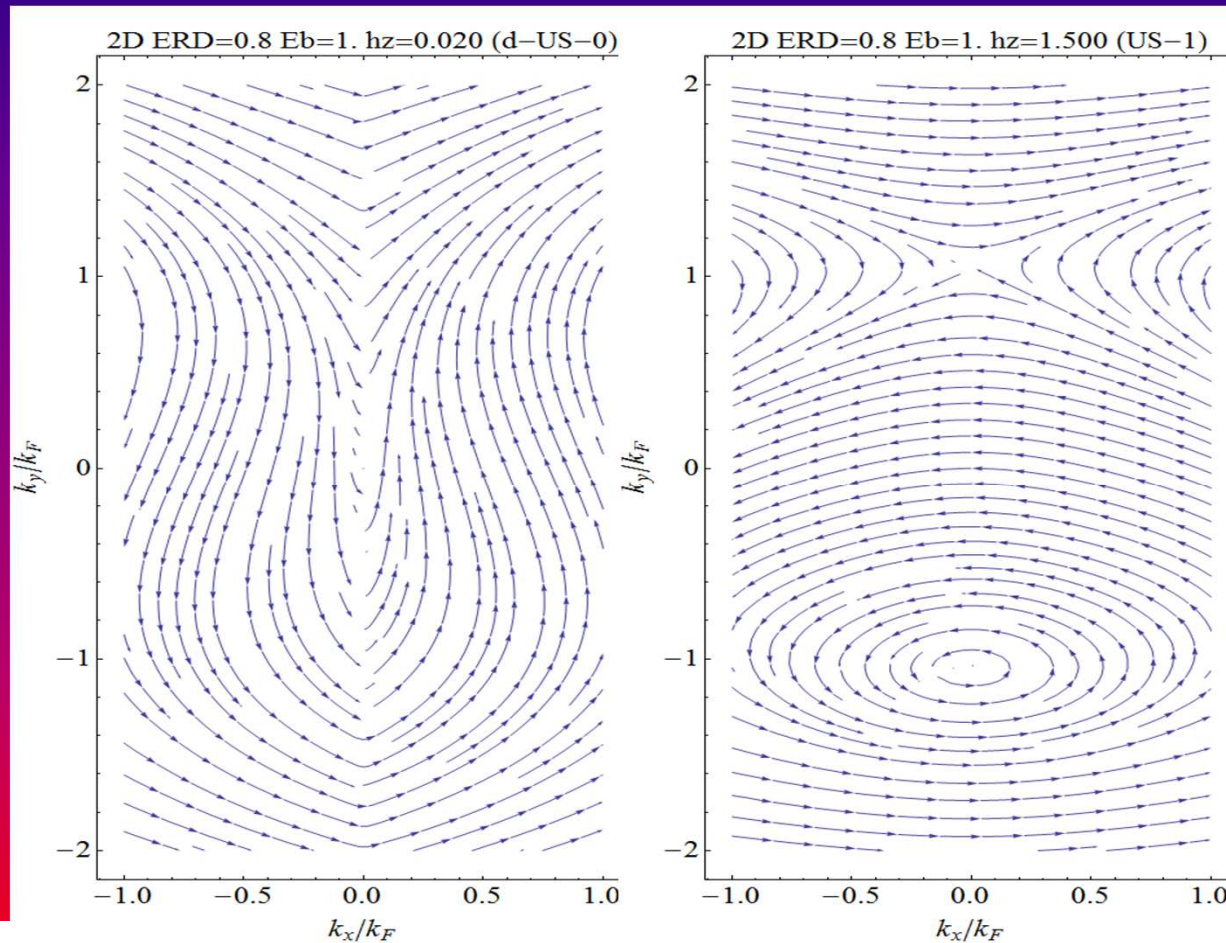
$$N_w = (2\pi)^{-1} \oint d\ell \hat{\mathbf{z}} \cdot \hat{\mathbf{m}} \times d\hat{\mathbf{m}}/d\ell$$

$$m_x(\mathbf{k}) = [E_S(\mathbf{k}) - |\mathbf{h}_{\text{eff}}(\mathbf{k})|] / E_{p-}(\mathbf{k})$$

$$m_y(\mathbf{k}) = \Delta_T(\mathbf{k}) / E_{p-}(\mathbf{k})$$



# Vortices and Anti-vortices of $m(\mathbf{k})$



$$h_z / \varepsilon_F = 0.2$$

US - 0

$$E_b / \varepsilon_F = 1.0$$

$$h_z / \varepsilon_F = 1.5$$

US - 1

For  $T = 0$  phase diagram need chemical potential and order parameter

$$\Omega_0 = V \frac{|\Delta_0|^2}{g} - \frac{T}{2} \sum_{\mathbf{k}, j} \ln \{1 + \exp [-E_j(\mathbf{k})/T]\} + \sum_{\mathbf{k}} \bar{K}_+,$$

$$\bar{K}_+ = \left[ \tilde{K}_\uparrow(-\mathbf{k}) + \tilde{K}_\downarrow(-\mathbf{k}) \right] / 2$$

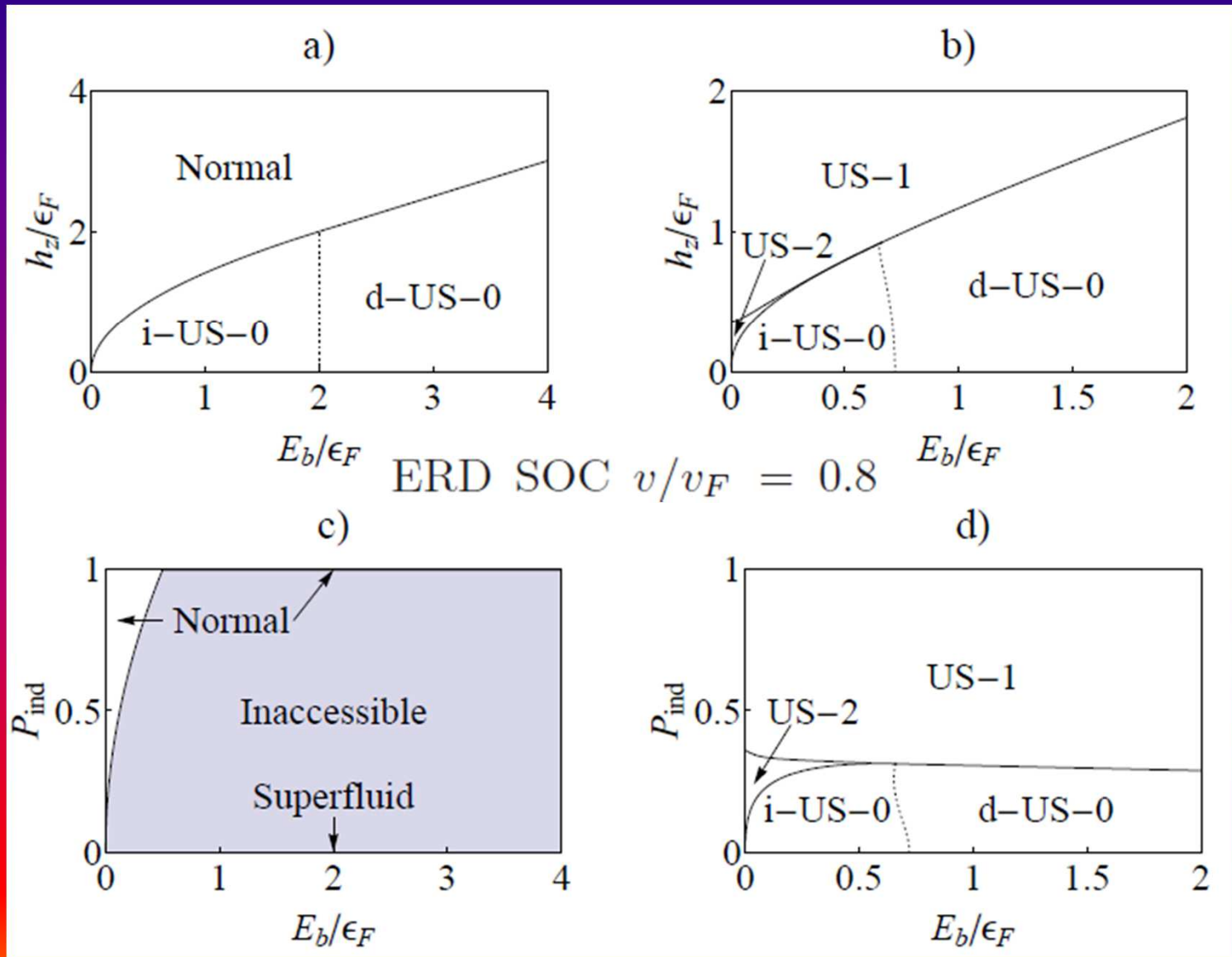
$$\frac{\delta \Omega_0}{\delta \Delta_0} = 0$$

Order  
Parameter  
Equation

$$N_+ = -\frac{\partial \Omega_0}{\partial \mu_+} = 0$$

Number  
Equation

# T = 0 Phase Diagram in 2D



# Momentum distributions in 2D

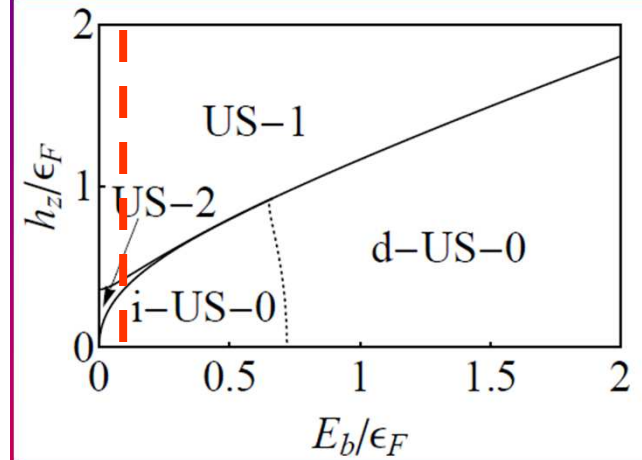
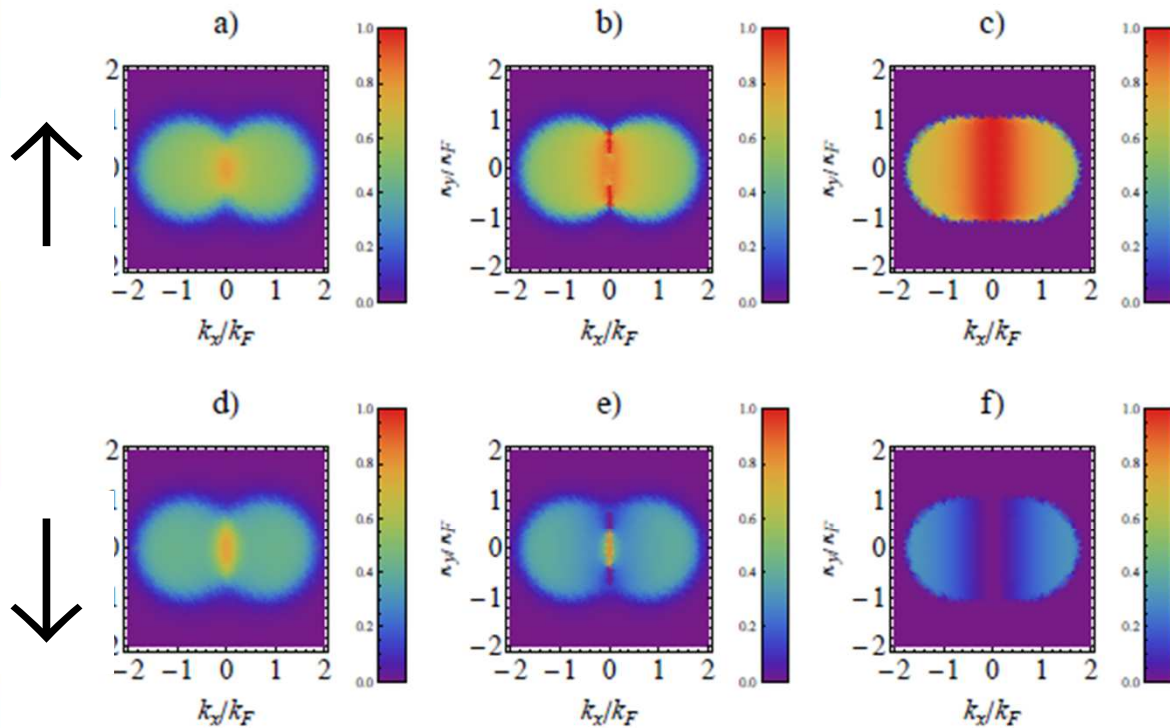
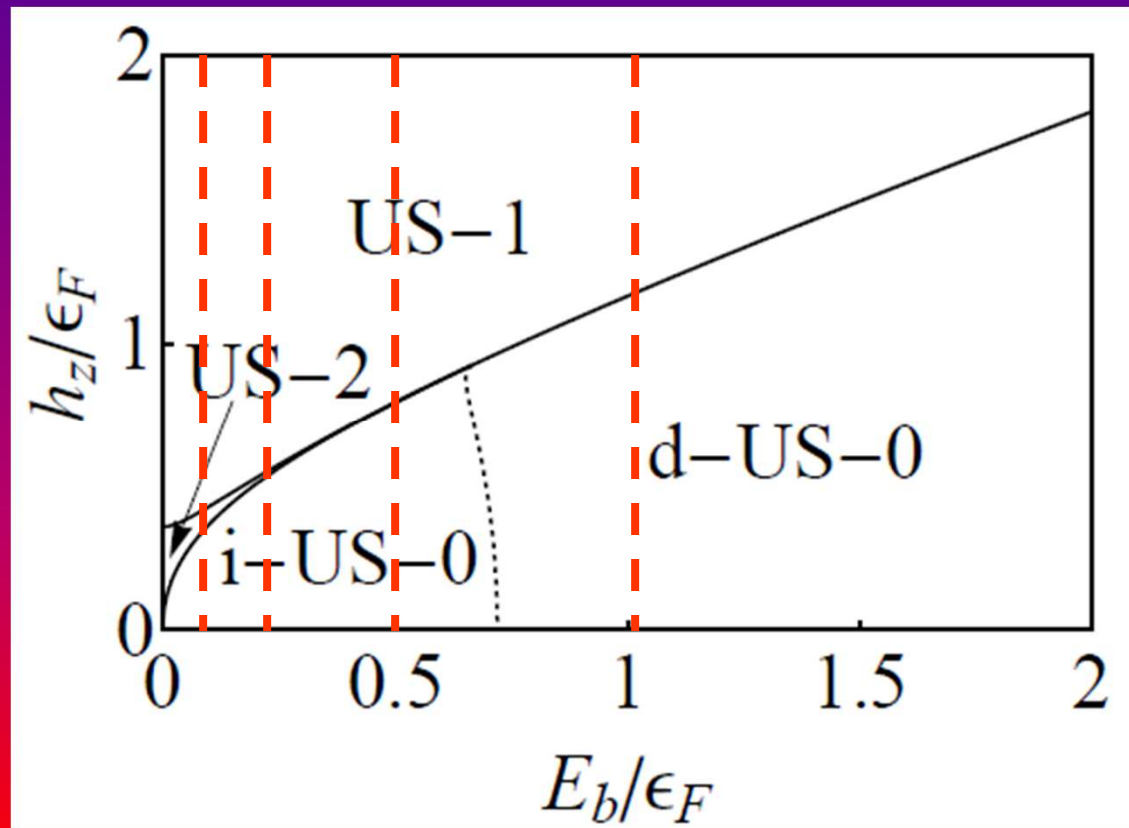


FIG. 3: (color online) The momentum distributions  $n_s(k_x, k_y)$  for ERD SOC  $v/v_F = 0.8$  and  $E_b/\epsilon_F = 0.1$  at  $T = 0$ , where  $s = \uparrow (\downarrow)$  for upper (lower) panels. (a)(d) i-US-0 phase with  $h_z/\epsilon_F = 0.2$ ; (b)(e) US-2 phase with  $h_z/\epsilon_F = 0.4$ ; (c)(f) US-1 phase with  $h_z/\epsilon_F = 1.0$ . The color coding varies continuously from purple ( $n_s = 0$ ) to red ( $n_s = 1$ ).

# Thermodynamic signatures of topological transitions



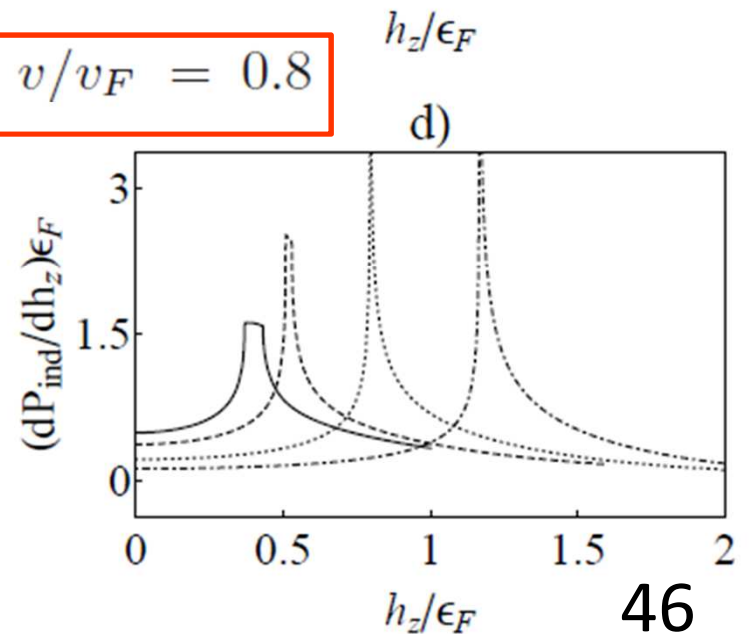
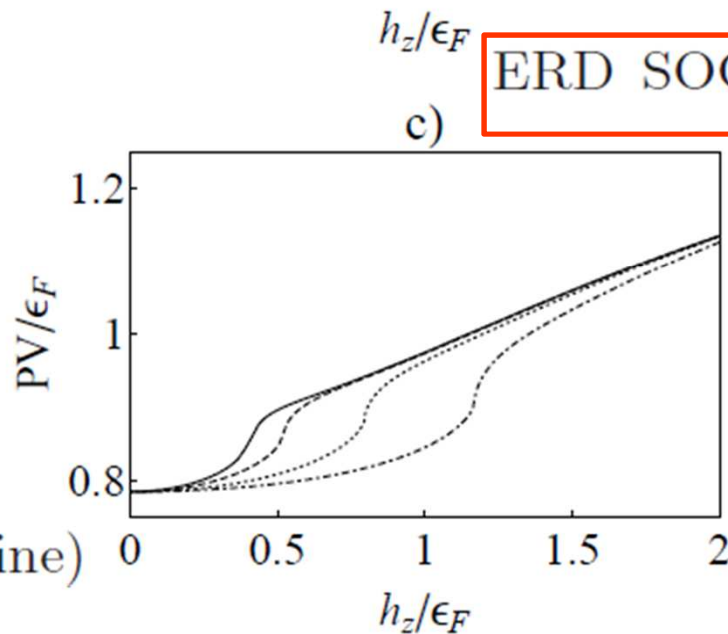
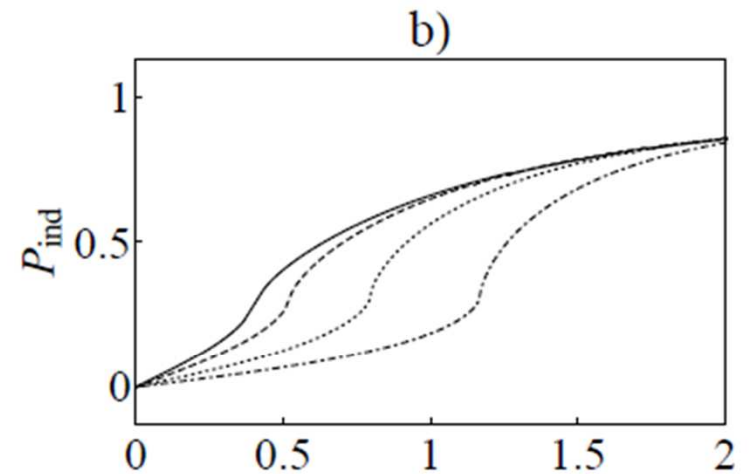
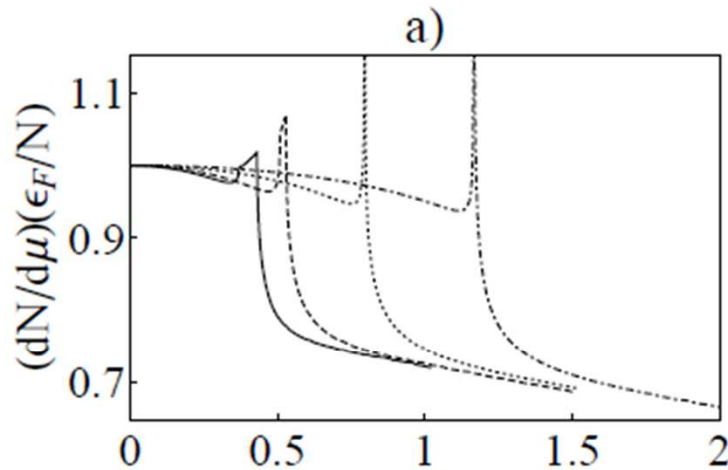
# T = 0 Thermodynamic Properties in 2D

$E_b/\epsilon_F = 0.1$   
(solid line)

$E_b/\epsilon_F = 0.2$   
(dashed line)

$E_b/\epsilon_F = 0.5$   
(dotted line)

$E_b/\epsilon_F = 1.0$   
(dot-dashed line)



ERD SOC  $v/v_F = 0.8$

# Outline

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# Hamiltonian in Real Space

$$\mathcal{H}(\mathbf{r}) = \mathcal{H}_0(\mathbf{r}) + \mathcal{H}_I(\mathbf{r})$$

$$\mathcal{H}_0(\mathbf{r}) = \sum_{\alpha\beta} \psi_{\alpha}^{\dagger}(\mathbf{r}) \left[ \hat{K}_{\alpha} \delta_{\alpha\beta} - h_i(\mathbf{r}) \sigma_{i,\alpha\beta} \right] \psi_{\beta}(\mathbf{r})$$

Kinetic Energy

Spin-orbit and Zeeman

$$\mathcal{H}_I(\mathbf{r}) = -g \psi_{\uparrow}^{\dagger}(\mathbf{r}) \psi_{\downarrow}^{\dagger}(\mathbf{r}) \psi_{\downarrow}(\mathbf{r}) \psi_{\uparrow}(\mathbf{r})$$

Contact Interaction



# Effective Action at finite T

$$\psi_{r,s} \rightarrow \psi_{r,s} e^{i\theta_r/2}$$

$$\Delta_r = |\Delta_r| e^{i\theta_r}$$

$$S = -\frac{1}{2} \text{Tr} \left\{ \ln \left[ \beta \begin{pmatrix} \mathbb{A}_+ & \mathbb{D}_+ \\ \mathbb{D}_- & \mathbb{A}_-^* \end{pmatrix} \right] \right\} - \frac{\beta L^2 |\Delta|^2}{g} \quad (2)$$
$$+ \frac{\beta}{2} \sum_{k,s} (-i\omega_n + \mathbf{k}^2 - \mu_s) + \frac{1}{8L^2} \int dr \sum_k [\nabla_r(\theta_r)]^2.$$

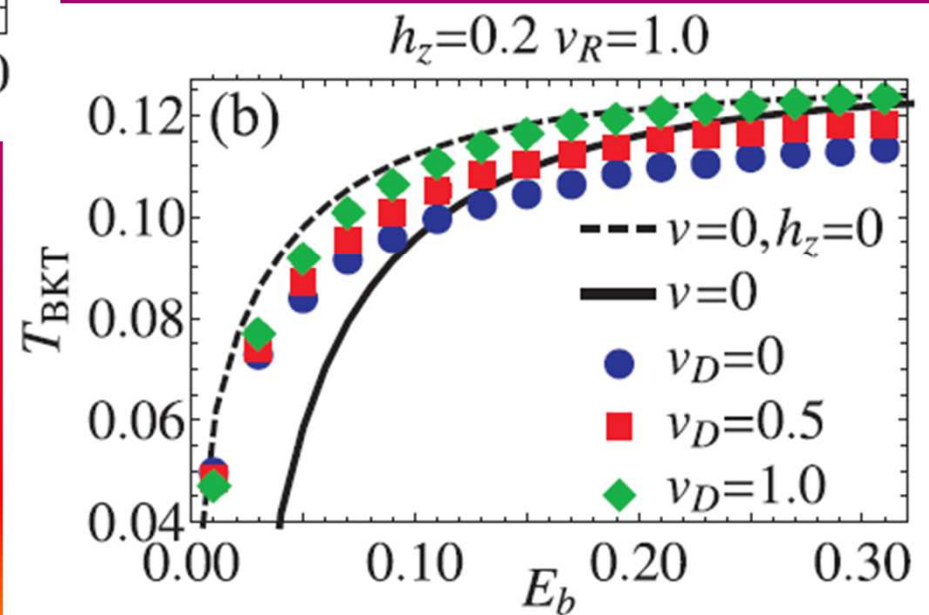
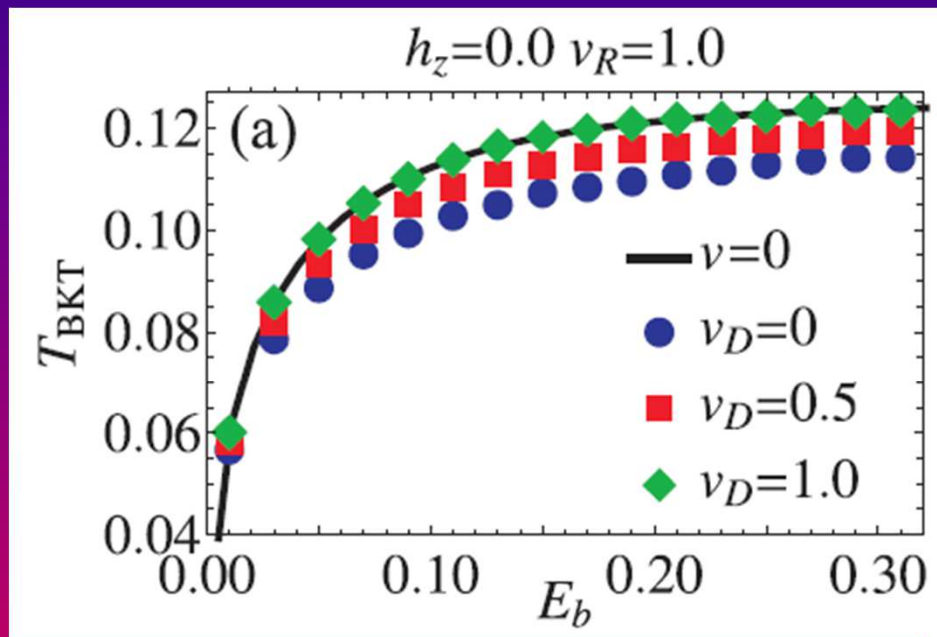
# Effective Action at finite T

$$S = S_{sp} + S_{fl}$$

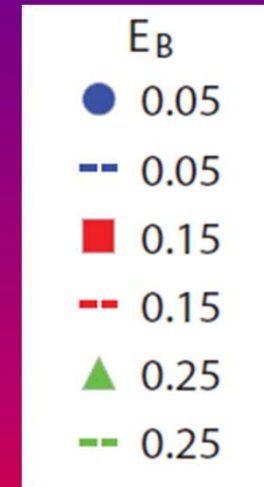
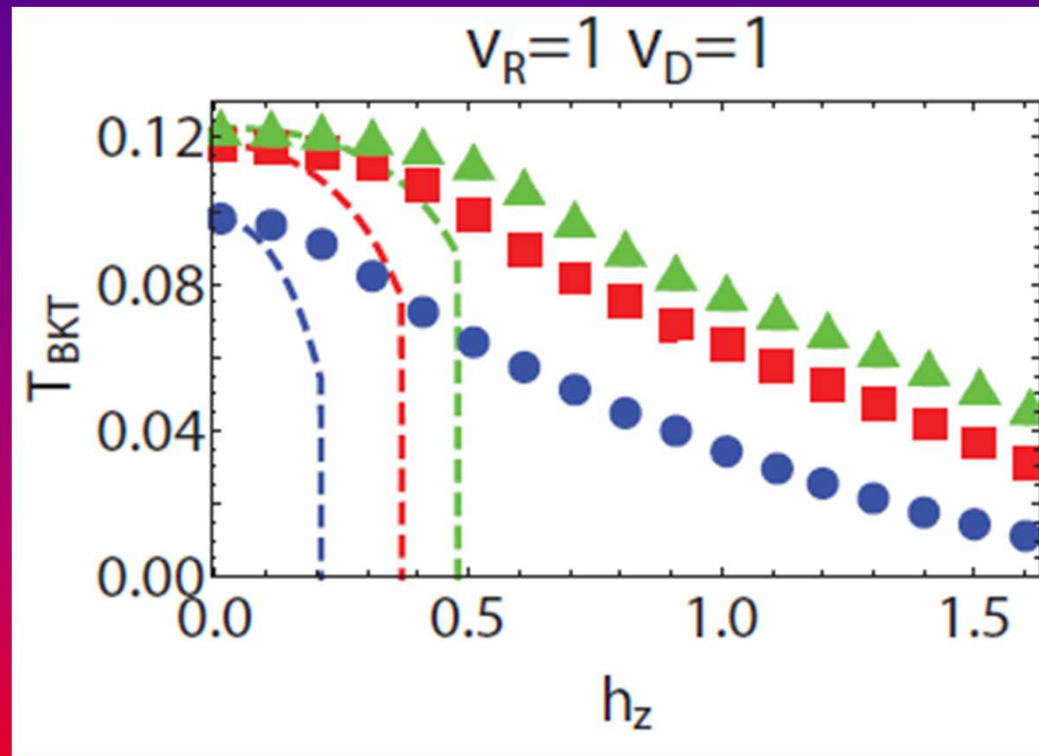
$$S_{sp} = -\frac{1}{2} \text{Tr} \{ \ln [\beta M_k(0, 0)] \} + \frac{\beta}{2} \sum_{k,s} (-i\omega_n + \mathbf{k}^2 - \mu_s) - \frac{\beta L^2 |\Delta|^2}{g}$$

$$S_{fl} = \frac{1}{2} \int dr \left( \mathcal{A} \left( \frac{\partial \theta_r}{\partial \tau} \right)^2 + \sum_{\nu=\{x,y\}} \rho_{\nu\nu} \left( \frac{\partial \theta_r}{\partial \nu} \right)^2 \right)$$

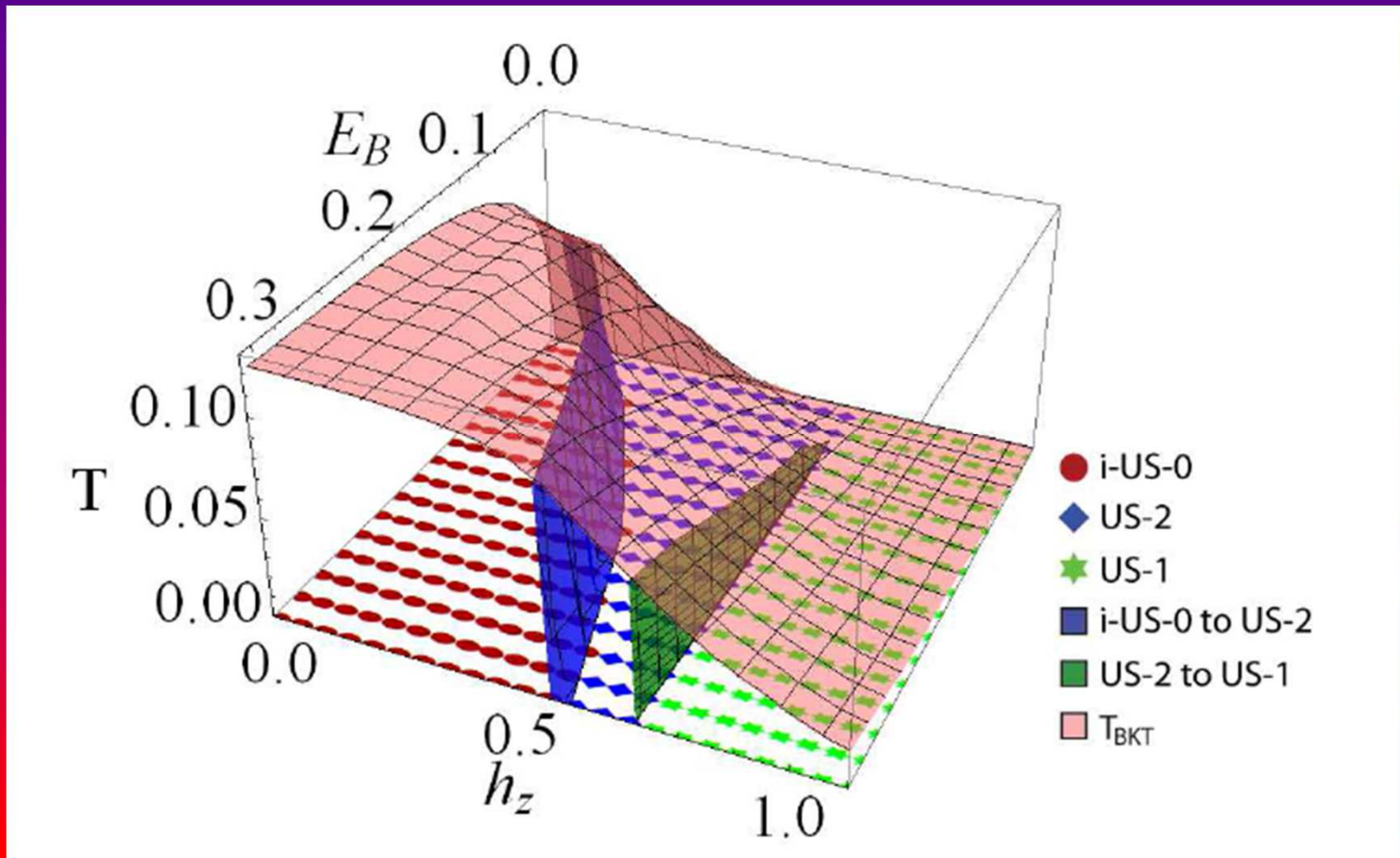
# BKT Transition Temperature



# Beyond the Clogston Limit

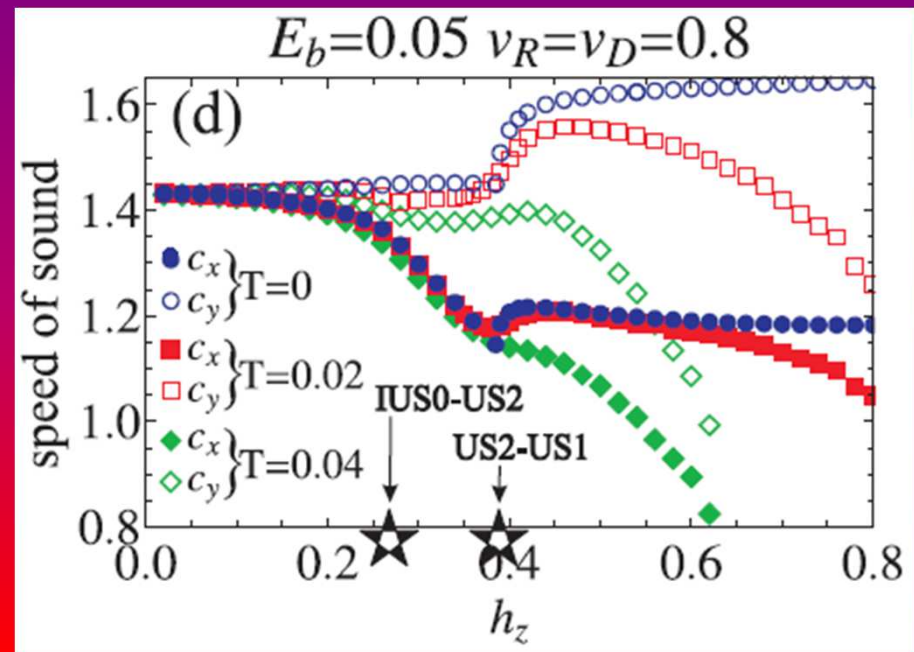
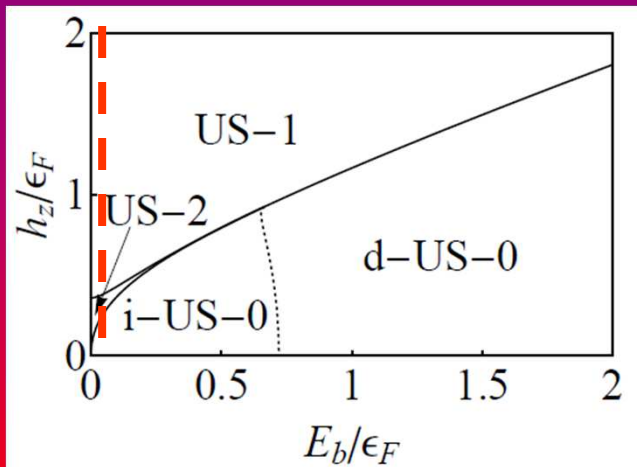


# Full Finite Phase Diagram



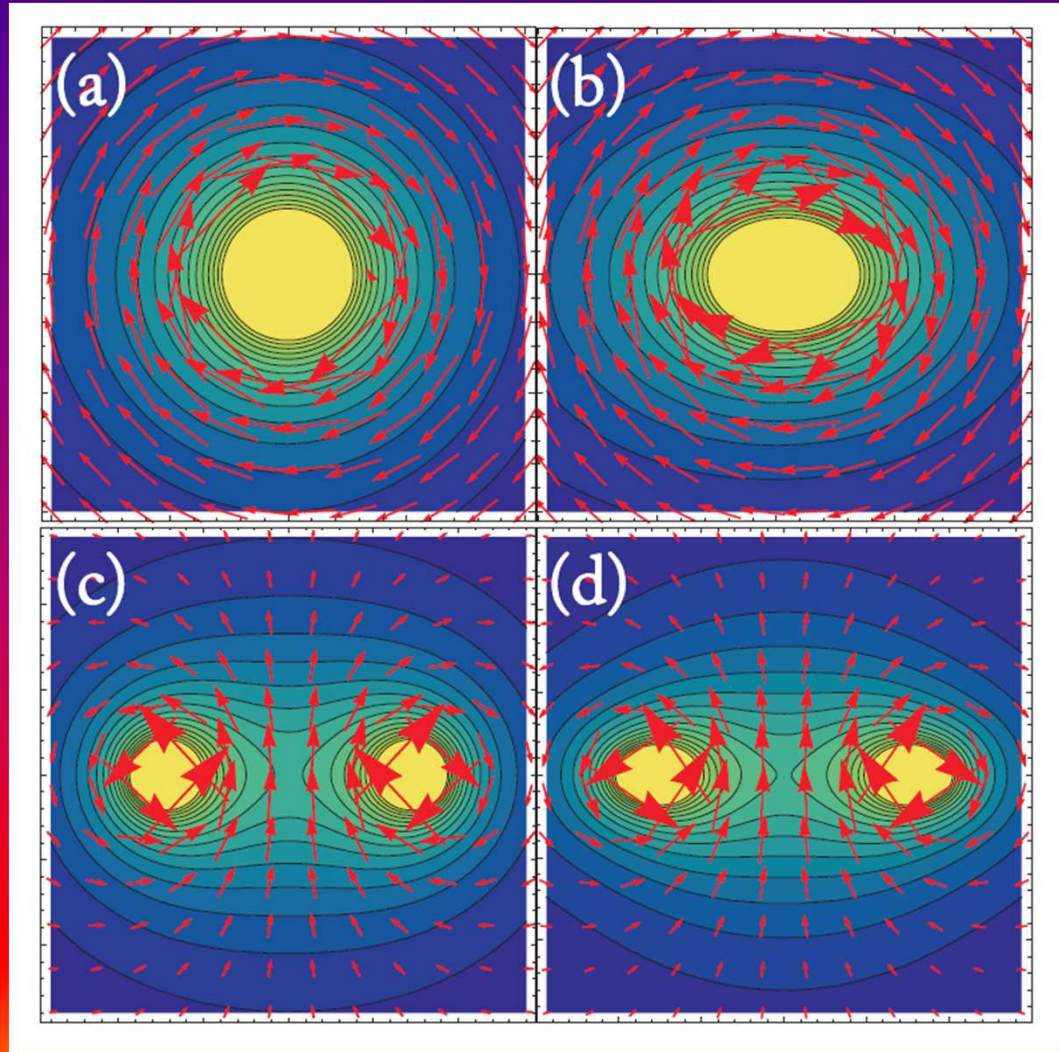
# Anisotropic speed of sound

$$\tilde{A}\omega^2 - (q_x \ q_y) \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{xy} & \rho_{yy} \end{pmatrix} \begin{pmatrix} q_x \\ q_y \end{pmatrix} = 0$$



# Vortex-Antivortex Structure

RASHBA



ERD

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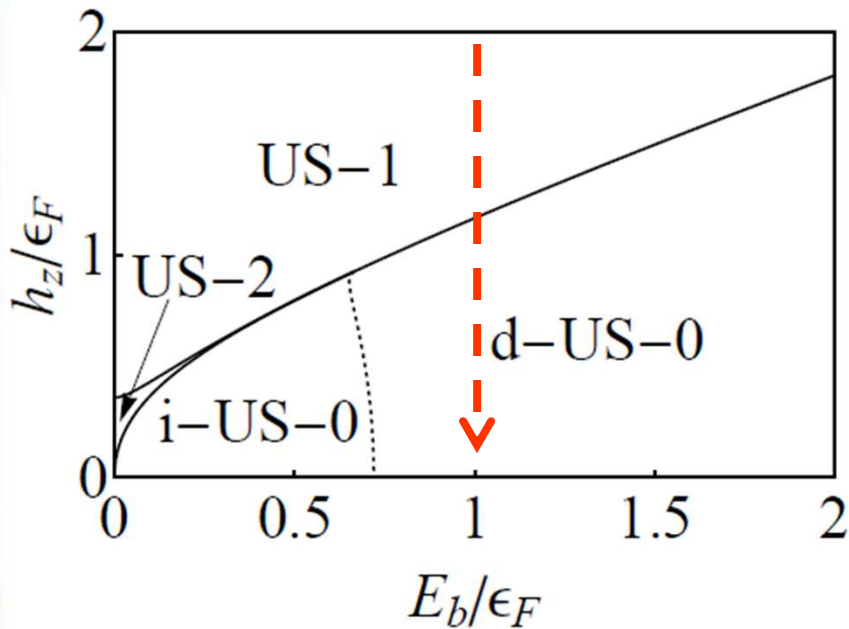
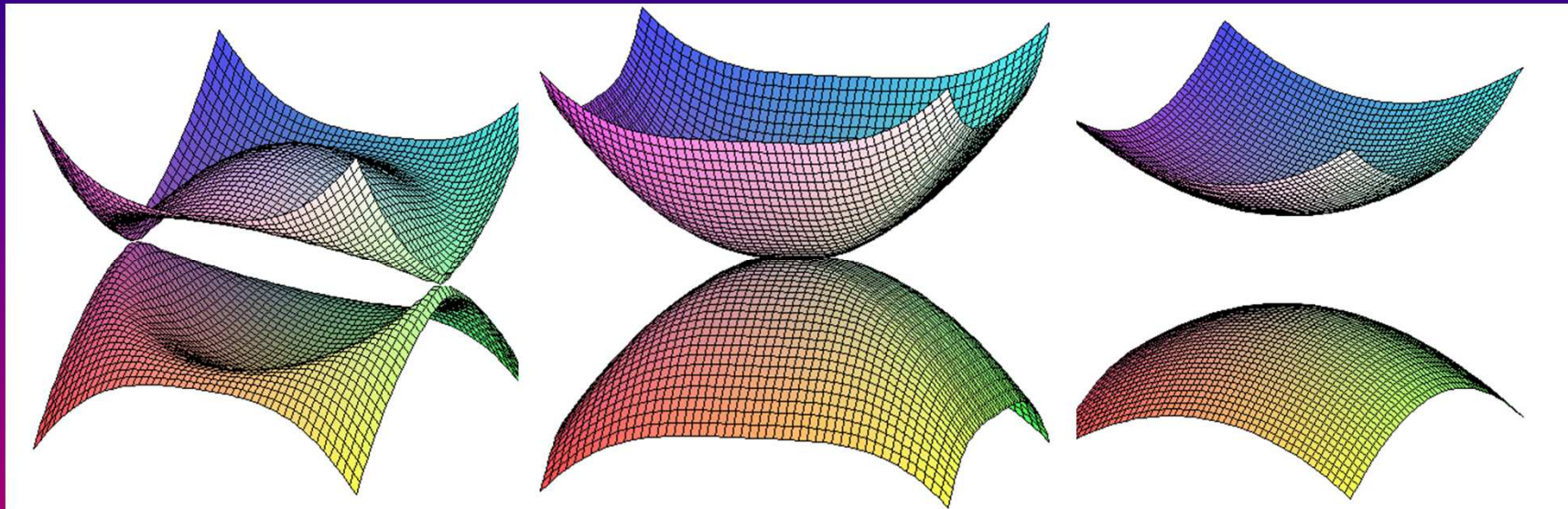
# Conclusions in words

- Ultra-cold fermions in the presence of spin-orbit and Zeeman fields are special systems that allow for the study of exciting new phases of matter, such as topological superfluids, with a high degree of accuracy.
- Topological quantum phase transitions emerge as function of Zeeman fields and binding energy for fixed spin-orbit coupling.

# Conclusions in words

- The critical temperature of the BKT transition as a function of pair binding energy is affected by the presence of spin-orbit effects and Zeeman fields. While the Zeeman field tends to reduce the critical temperature, SOC tends to stabilize it by introducing a triplet component in the superfluid order parameter.
- In the presence of a generic SOC the sound velocity in the superfluid state is anisotropic and becomes a sensitive probe of the proximity to topological quantum phase transitions. The vortex and antivortex shapes are also affected by the SOC and acquire a corresponding anisotropy.

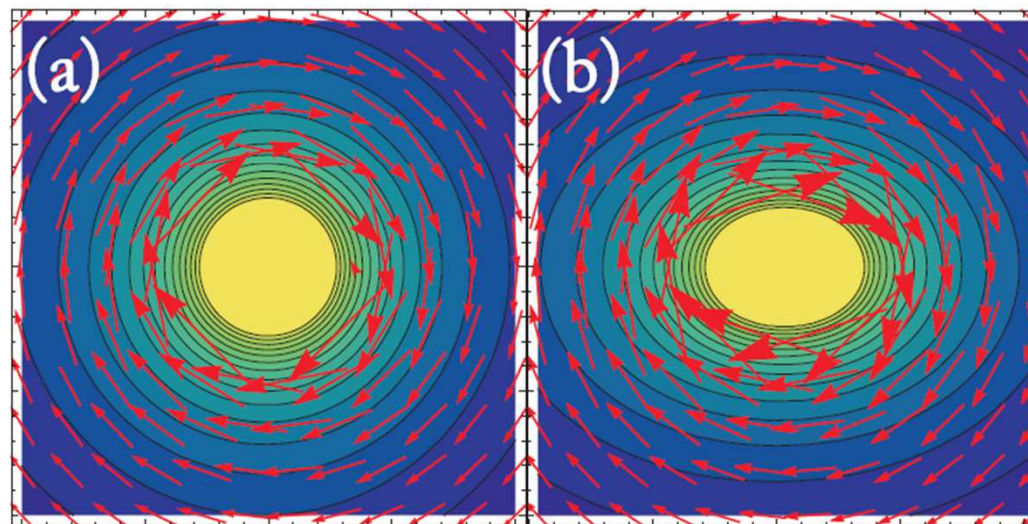
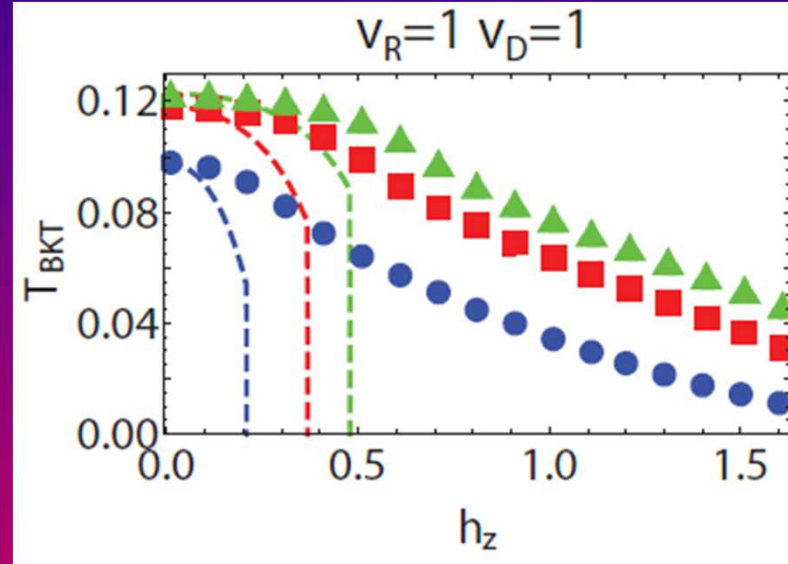
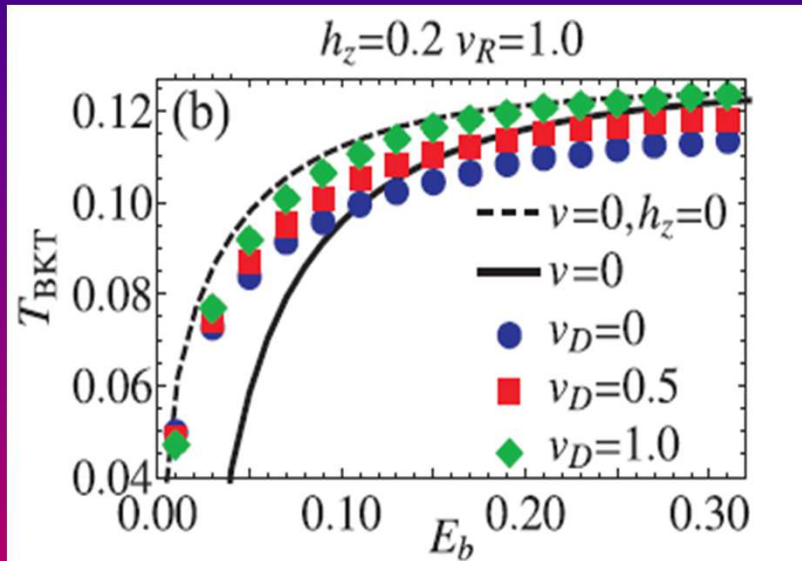
# Conclusions in Pictures



**Change in topology**

**TRANSITION FROM  
GAPLESS TO GAPPED SUPERFLUID**

# BKT transition and vortex-antivortex structure



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THE END